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REVIEW ARTICLE

Survey of theories of anomalous transport

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Abstract. Energy and particle confinement in tokamaks is usually anomalous, greatly exceeding neoclassical predictions. It is desirable to develop an understanding of the underlying processes to increase our confidence in extrapolation of tokamak behaviour towards reactor regimes. The literature abounds with theoretical expressions for anomalous transport coefficients based on turbulent diffusion due to various micro-instabilities. These often purport to provide explanations of tokamak confinement at the level of global scaling laws. However, comparison with experimental data from local transport analyses offers a far more stringent test of these theories. This review presents the available theories for turbulent transport coefficients, particularly ion and electron thermal diffusivities, in a way that will facilitate a programme of testing models against data. It provides a brief description of the basis for each theory to place it in context and then presents the resulting turbulent diffusivity. Particular emphasis is placed on the validity conditions under which the expressions may be used; this is important when subjecting them to meaningful tests against data. The present review emphasizes the more recent developments, building on earlier ones by Liewer and Ross *et al*. The results of this work have already been of value in carrying out a programme of testing theories against high quality JET data (Connor *et al* and Tibone *et al*).

1. Introduction

The plasma physics literature abounds with theoretical expressions for anomalous transport coefficients (see the earlier review by Liewer 1985) purporting to explain the confinement properties of tokamaks, often on the basis of crude comparisons at the level of energy confinement time scalings. The availability of detailed profile data on transport coefficients from machines such as JET in a range of relevant and varied regimes provides an opportunity to undertake a systematic comparison of these theories with experiment. As a first step to realizing this we review the literature on anomalous transport and collect together published transport coefficients, with an emphasis on the more recent ones. This extends and updates the similar exercise of Ross (1987).

The vast majority of these transport coefficients are based on turbulent transport due to fluctuations on a microscopic scale length such as the ion Larmor radius, collisionless skin depth or resistive layer width. Consequently they can be cast in the generic gyro-Bohm form (Connor and Taylor 1985; Hagan and Frieman 1986; Connor 1988)

$$\chi_{e,i} = \frac{c_s \rho_s^2}{L_n} F_{e,i}(\nu_{*e}, \beta, \tau, m_e/m_i, q, s, \eta_i, \eta_e, \epsilon_n \ldots) \tag{1.0.1}$$

where ν_{*e} is the electron collisionality, β is the ratio of thermal to magnetic energy, τ is the ratio of electron to ion temperature, m_j is the species mass, q is the safety factor, s is the magnetic shear, $\eta_j = |\nabla \ln T_j|/|\nabla \ln n_j|$ (where T_j and n_j are the species temperature and density), $L_n^{-1} = |\nabla \ln n|$, $\epsilon_n = L_n/R$ and R is the major radius. We have also defined the sound speed, $c_s = \sqrt{T_e/m_i}$ and the ion Larmor radius evaluated at the sound speed, $\rho_s = \sqrt{m_i T_e}/eB$. In practical units (i.e. temperature in keV, magnetic field in Tesla, length scales in metres) we have

$$\chi_{e,i} = 3.23 \frac{\mu^{1/2} T_e^{3/2}}{B^2 L_n} F_{e,i} \tag{1.0.2}$$

for the diffusivity (in $m^2 s^{-1}$) where we have defined μ as the ratio of the ion to proton mass. Equation (1.0.1) is to be compared with the Bohm form

$$\chi_{e,i} \sim \rho_s c_s F'_{e,i} \tag{1.0.3}$$

which is associated with transport due to longer wavelength structures whose scale is related to the plasma minor radius.

Since the information which we collect is to be useful for comparison with experimental data we take care to define the validity conditions for the applicability of the various theories and to express the results in a form convenient for evaluation. If the transport coefficients are large once an instability condition is exceeded the plasma profiles may then sit at marginal-stability; stability criteria are therefore of value for comparing with experiment and attention is also drawn to them. In addition to compiling the expressions we comment in some detail on their physical and theoretical basis in order to provide understanding and allow an assessment of their value. Some complementary aspects have been addressed recently by Horton (1990).

The structure of the review is as follows. First, theories of ion transport, essentially those involving ion temperature gradient turbulence, are described and discussed in section 2. After a brief overview in section 2.1 we discuss the slab, toroidal and trapped-ion ∇T_i modes in sections 2.2, 2.3 and 2.4 respectively, adding some conclusions in section 2.5. Treatments of electron transport due to electrostatic and electromagnetic drift-wave turbulence follow in section 3. In section 3.1 we provide an overview and then discuss general transport coefficients based on the assumption that drift-wave turbulence exists in the tokamak plasma without specific reference to its source (in section 3.2). We then consider specific instabilities which may be responsible for driving the electrostatic (in section 3.3) and electromagnetic (in section 3.4) turbulence. Electrostatic modes considered include the 'universal' mode and trapped electron modes. In the electromagnetic drift-wave subsection we consider the electron drift wave, the drift micro-tearing mode and the η_e mode. Some general conclusions are drawn in section 3.5. In section 4 we consider the transport due to magnetic islands which arise from nonlinear instabilities. After an overview in section 4.1, the various drives due to bootstrap currents, drift effects, etc are described in section 4.2; some concluding remarks are made in section 4.3. Finally, in section 5, we consider theories of transport due to magneto-hydrodynamic (MHD) turbulence with an Ohm's law including resistivity (incorporating neoclassical effects) or electron inertia. Section 5.1 provides an overview and section 5.2 addresses the fluid pressure-gradient driven modes (e.g. resistive ballooning modes) while section 5.3 considers resistivity-gradient modes; some conclusions are in section 5.4. Some overall concluding remarks are made in section 6.

Although in principle we would wish to include the whole matrix of transport coefficients these are rarely all given. The emphasis, therefore, is on electron and ion thermal diffusivities but other coefficients are listed when possible. For ease of use these are also presented in tabular form in appendix B; a table of the notation used is given in appendix A.

The results of this survey have been used by JET and Culham colleagues, in a comparison of theories of anomalous transport with JET data and the results are reported in Connor *et al* (1993) and Tibone *et al* (1994).

2. Ion transport due to ∇T_i instabilities

2.1. Overview

We begin with a brief overview of earlier work on the ion heat transport which might be expected to result from instabilities driven by an ion temperature gradient; this sets the scene for the discussion of recent work which follows in sections 2.2, 2.3 and 2.4. Such modes are often characterized by a value of η_i (where η_i is the ratio of the density length scale to that of the temperature) and are therefore often referred to as 'η_i modes'. However, in the limit of a flat density profile ($\eta_i \to \infty$) the mode is characterized by a critical temperature gradient, and hence the mode is also called the ∇T_i or ion temperature gradient (ITG) driven mode. A whole family of these modes exists depending on the tokamak conditions.

The most basic version of the ∇T_i instabilities is the slab mode, which occurs as a result of ion acoustic waves coupling to a radial gradient in the ion pressure. The two fluid dispersion relations for the electrostatic modes (of frequency ω and wavenumber $k_\parallel$ along the magnetic field and k_y perpendicular to it) in a shearless slab takes the form (Horton and Varma 1972)

$$\omega^3 - \omega^2\omega_{*e} - k_\parallel^2 c_s^2 \left[\omega + \frac{1}{\tau}\left(\frac{5}{3}\omega + \omega_{*e}\left(\eta_i - \frac{2}{3}\right)\right)\right] = 0 \tag{2.1.1}$$

where $\omega_{*e} = k_y\rho_s c_s/L_n$ is the electron diamagnetic frequency. In the limit $\eta_i \gg 1$ this gives rise to the unstable solution

$$\omega = \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)\left(k_\parallel^2 c_s^2 \eta_i \omega_{*i}\right)^{1/3} \tag{2.1.2}$$

the so-called η_i mode. Consideration of Landau damping and finite Larmor radius (FLR) in a kinetic description yields a critical value of η_i for instability given by

$$\eta_{ic} \approx 0.95 \tag{2.1.3}$$

achieved for $k_y\rho_s \simeq 1$. Inclusion of shear leads to a radial eigenvalue problem and the growth rate depends on the radial mode number l. In particular, the maximum growth rate occurs for $(k_y\rho_s)^2 = \tau(1+\eta_i)^{-1}$ and is given by

$$\frac{\gamma}{\omega_{*e}} \sim \sqrt{\frac{L_n}{L_s}\frac{(1+\eta_i)}{\tau}(2l+1)} \tag{2.1.4}$$

for $L_n/L_s \ll 1$ (e.g. Lee and Diamond 1986).

The turbulence driven by the η_i mode is expected to give rise to ion heat transport. Scale invariance properties of the equations describing the mode (Connor 1986a) indicate that the ion heat diffusivity, χ_i, must take the form

$$\chi_i \sim \frac{\rho_s^2 c_s}{L_n} F\left(\frac{L_n}{L_s}, \frac{L_n}{L_T}\right). \tag{2.1.5}$$

One of the earliest calculations of the heat diffusivity resulting from η_i mode turbulence was made by Kadomtsev and Pogutse (1970). The effects of the turbulence are modelled by inserting an effective heat diffusivity, χ_i, into the ion drift kinetic equation and solving for the radial eigenmode structure. An expression for χ_i is determined from the condition that it should render the most dangerous mode stable, thus

$$\chi_i \sim \frac{1}{40}\frac{\rho_s^2 c_s}{L_T}\frac{L_s}{L_T} \tag{2.1.6}$$

which is indeed of the form predicted by the scaling arguments. We shall see later that in subsequent work by Lee and Diamond (1986) (using turbulence theory) and Connor (1986a) (using an extension of the scale invariance properties) a form for χ is derived which increases with shear. However, numerical simulation by Hamaguchi and Horton (1990) indicates a positive dependence on L_s. The discrepancy is associated with an approximation in the Lee and Diamond (1986) and Connor (1986a) works and the majority of subsequent improved calculations predict χ_i to increase with L_s. Terry *et al* (1988) considered the higher radial eigenmode numbers, l and found that while $\chi \sim 1/L_s$ for $l = 0$, for higher l, χ_i has a positive scaling with L_s, in agreement with numerical simulation. Furthermore, the transport is predicted to be dominated by the higher l eigenmodes as a result of their greater radial mode width.

Horton *et al* (1981) and Guzdar *et al* (1983) consider the mode in toroidal geometry where it is found that unfavourable curvature replaces the acoustic wave as the main driving mechanism. The mode then has a different spatial structure and becomes more 'ballooning' in nature. Using a fluid picture (e.g. Horton *et al* 1981) one finds modes poloidally localized in the region of unfavourable curvature with a maximum growth rate

$$\frac{\gamma}{\omega_{*i}} \sim \sqrt{2\epsilon_n(1+\eta_i)}\,. \tag{2.1.7}$$

The threshold in η_i is determined by Landau drift resonances (Romanelli 1989):

$$\eta_{ic} = \begin{cases} 1 & \epsilon_n < 0.2 \\ 1 + 2.5(\epsilon_n - 0.2) & \epsilon_n > 0.2\,. \end{cases} \tag{2.1.8}$$

A quasilinear estimate of the resulting ion heat transport (Horton *et al* 1981) yields

$$\chi_i \sim \frac{\rho_s^2 c_s}{L_n}\frac{q}{s}(1+\eta_i)^{1/2}\,. \tag{2.1.9}$$

The two distinct mode structures lead to a natural categorization of the instability into either 'slab' or 'toroidal'. As shown in the work by Horton *et al* (1981) the theories apply in two different regions of parameter space. In particular,

$$\begin{aligned} L_s &< \frac{R}{2} \Rightarrow \text{slab theory applies} \\ L_s &> \frac{R}{2} \Rightarrow \text{toroidal theory applies}\,. \end{aligned} \tag{2.1.10}$$

This may lead to complications when comparing theoretical predictions with a given tokamak because at the centre (where the shear length is typically very long) a toroidal theory applies and towards the edge (with shorter shear lengths) slab theories may be more applicable.

Consideration of toroidicity introduces a further complication—the trapped particles which exist as a consequence of the inhomogeneity in the magnetic field. For long-wavelength perturbations such that $\omega \lesssim \omega_{*i} < \omega_{bi}$ (where ω_{*i} and ω_{bi} are the ion diamagnetic frequency and bounce frequency respectively) trapped and passing particles have different behaviours (Kadomtsev and Pogutse 1971) and this gives rise to a new class of instabilities, called the trapped-ion modes, which can be driven unstable by the ion temperature gradient. These modes can be categorized according to the value of η_i (Biglari *et al* 1989). For $\eta_i < \frac{4}{3}$ ion collisions are stabilizing and the mode is destabilized by electron collisions. Kadomtsev and Pogutse (1971) analyse this mode using a model involving trapped and passing electron and ion fluids. In the limit $\nu_e/\epsilon \gg \sqrt{\epsilon}\omega_{*e}$ they derive the following complex mode frequency for the long-wavelength 'dissipative' trapped-ion instability:

$$\omega = \frac{\sqrt{\epsilon}\omega_{*e}}{1+\tau} - \mathrm{i}\frac{\nu_i}{\epsilon} + \mathrm{i}\frac{\epsilon^2}{(1+\tau)^2}\frac{\omega_{*e}^2}{\nu_e}. \tag{2.1.11}$$

For increasing collisionality the mode is stabilized. In fact, inclusion of passing ion Landau damping in the analysis leads to the stability criterion (Kadomtsev and Pogutse 1971)

$$\frac{\nu_i}{\omega_{bi}} > 0.52\epsilon\left(\frac{m_e}{m_i}\right)^{1/3}\tau|m-nq| \tag{2.1.12}$$

where m and n are chosen so that $|m-nq| \ll 1$. Using $\chi_i \sim \sqrt{\epsilon}\gamma/k_\perp^2$ (where the $\sqrt{\epsilon}$ factor represents the fraction of trapped ions) yields

$$\chi_i \sim \epsilon^{5/2}\frac{\rho_s^2 c_s^2}{\nu_e L_n^2}. \tag{2.1.13}$$

As η_i is increased above the value of $\frac{4}{3}$ the mode characteristics alter. The most significant change is that ion collisions now become destabilizing, tapping the energy source of the ion pressure gradients (Biglari *et al* 1989). Finally, at large η_i ($\gtrsim O(\epsilon^{-1/2})$) the mode becomes independent of particle collisions and is fluid-like in nature.

We consider three cases of the ∇T_i instability (slab, toroidal and trapped ion) in the following three subsections (respectively). In each case we will discuss the stability thresholds and give expressions for the thermal diffusivity. As noted in the introduction, these have the form

$$\chi_i = 3.23\frac{\mu^{1/2}T_e^{3/2}}{L_n B^2}F(\eta_i, \tau, s, q, \epsilon_n, \ldots) \tag{2.1.14}$$

where F is a function of the dimensionless variables of the equilibrium. We shall therefore quote values for F in this section, rather than χ_i. The results given are for a plasma with a singly charged ion species with no impurity present. Impurities can be taken into account by following the prescription derived by Mattor (1991) who demonstrates that one should replace the ion density scale length with that of the electrons.

2.2. *Slab* ∇T_i *mode*

We now consider recent developments in the theory of the ∇T_i mode in a slab geometry. Before transport due to this mode can be addressed one should first determine stability and this is governed by the value of η_i. For large η_i ($> \eta_c$) the mode is unstable and there are several theories which calculate η_c. As noted in (2.1.3) earlier collisionless theories found

$$\eta_c \simeq 0.9 \tag{2.2.1}$$

but several, more recent, theories obtain modifications to this. Thus work by Hassam *et al* (1990) shows, using a fluid treatment, that very long wavelength modes ($k_\parallel v_{thi} < \nu_{ii}$) behave collisionally close to marginal stability, and the threshold is lowered to

$$\eta_c = \tfrac{2}{3}. \tag{2.2.2}$$

The above two results imply that flat density profiles (such as those observed in H-mode plasmas) should be very unstable. This is apparently in contradiction with the observed good confinement properties of H-mode plasmas. However, when $L_n/L_s \gtrsim 1$ the instability criterion becomes a condition on ∇T_i rather then η_i. For example, in their analysis of flat density profiles Hahm and Tang (1989) find that the critical temperature gradient for stability is

$$\left(\frac{1}{L_{T_i}}\right)_c = \frac{3}{2}\sqrt{\frac{\pi}{2}}(1+\tau^{-1})\frac{(2l+1)}{L_s} \tag{2.2.3}$$

where l represents the radial mode number which has been excited. In the case of an inverted density profile (i.e. one in which the density increases towards the edge) Hahm and Tang find that for instability one requires $\eta_i < \eta_c$, where

$$\eta_c = -(1+\tau)\,. \tag{2.2.4}$$

The use of a fluid treatment to obtain this result is justified as the real part of the mode frequency does not go to zero at marginal stability.

Using weak-turbulence theory it has been shown (Mattor and Diamond 1989) that close to the threshold for linear stability the value of χ_i is much lower than would be obtained from an extrapolation of the results of strong turbulence theories (valid at much higher η_i). This result leads to the conclusion that it is not the threshold for linear stability which is important; rather one should use the threshold for strong turbulence (Mattor 1989). This leads to a slightly higher 'effective' threshold of

$$\eta_c \simeq 1.2 \tag{2.2.5}$$

which might be more relevant for a marginal-stability condition.

Let us now turn to the ion thermal conductivities which are predicted to exist if the plasma is unstable to the slab-like η_i mode. Early work of Connor (1986a) and Lee and Diamond (1986) obtain expressions for χ_i which increase with the 'shear' parameter L_n/L_s. In particular, Connor (1986a) uses the scale invariance properties of the fluid equations describing the η_i mode in the limit $L_n/L_s \ll 1$, $\eta_i L_n/L_s \gg 1$ to derive

$$F = g\frac{L_n}{L_s}\eta_i^{3/2} \tag{2.2.6}$$

where g is a numerical factor. Lee and Diamond (1986) explicitly calculate saturated turbulence levels in the same limits as those considered by Connor. From these they derive

$$F = [C(\eta_i)]^2[(1+\eta_i)/\tau]^2(k_y\rho_s)\frac{L_n}{L_s} \tag{2.2.7}$$

where $C(\eta_i) = (\pi/2)\ln(1+\eta_i)$. Taking $k_y\rho_s = 0.3$ and bearing in mind that $C(\mathrm{Re})$ varies slowly with η_i, this result is very similar to that obtained by Connor.

Subsequent numerical analysis of the fluid equations by Hamaguchi and Horton (1990) shows that an inertial term dropped by Connor (1986a) and Lee and Diamond (1986) can be important. In fact, using numerical simulation to retain the terms which were dropped in the above two calculations gives a *decrease* in χ_i as L_n/L_s is increased. In particular, they derive

$$F = \frac{1}{\tau}(\eta_i - \eta_{ic})\exp\left(-5\frac{L_n}{L_s}\right) \tag{2.2.8}$$

by a careful scanning of the parameter space. The dependence on L_s is taken from the numerical solution. However, the choice of the linear scaling with $\eta_i - \eta_c$ is biased by a mixing-length theory that was also described in this work. Although their numerical work is not in contradiction with this scaling, it does show that higher powers of $\eta_i - \eta_c$ (up to about 1.5) could produce a tolerable fit. The authors state that one can also fit the numerical results with an $(L_n/L_s)^{-p}$ scaling (with $p = \frac{1}{2}$ for $L_n/L_s < 0.1$ and $p = 2$ for $L_n/L_s \gtrsim 0.5$).

As discussed in the overview, Terry *et al* (1988) show that a positive scaling of χ with L_s is obtained by considering the higher-order radial eigenmodes. Furthermore, these higher-order modes are predicted to dominate the transport.

Following Hamaguchi and Horton, this decrease of χ_i with shear is a common feature of subsequent heat diffusivity calculations. For example, using gyrokinetic equations and mixing-length estimates of the transport, Mattor (1989) calculates values for χ_i in the limits $\eta_i \gtrsim \eta_c$ and $\eta_i \gg \eta_c$. Using the trends from these two results and numerical studies, the following fit for F is derived:

$$F = 0.037\left(\frac{L_s}{L_n}\right)^2\left(\frac{\tau}{1+\tau}\right)^{3/2}(\eta_i - 1.2)^3 . \tag{2.2.9}$$

This gives a stronger η_i dependence than that predicted by Hamaguchi and Horton (1990) but the dependence on the shear is consistent with their alternative (power-law) shear scaling (for $L_n/L_s \gtrsim 0.5$).

Hassam *et al* (1990) investigate the properties of the η_i mode close to threshold. As discussed above, the plasma appears collisional for long-wavelength modes ($k_\parallel v_{th\,i} < \nu_{ii}$) and there exists a lower threshold than for the shorter-wavelength modes. It is found that these long-wavelength modes contribute relatively little to the transport in the region $\frac{2}{3} < \eta_i < 0.9$ (i.e. the region unstable to long-wavelength modes but still stable to the short-wavelength modes). Above $\eta_i = 0.9$ the short-wavelength modes become unstable and the transport increases but is still relatively low up to $\eta_i \sim 2$. Once η_i exceeds 2 the transport becomes very large indeed. This is similar to the results of Mattor and Diamond (1989) obtained from a weak turbulence theory close to threshold. Using mixing-length arguments the following expression is derived for these long-wavelength (collisional) modes when $L_n/L_s \ll 1$:

$$F = h(\eta_i)\frac{\tau^{-1}L_s^2}{L_n^2}\left(\frac{\nu_{ii}L_n}{c_s}\right) \qquad 2 > \eta_i > \tfrac{2}{3} \tag{2.2.10}$$

where ν_{ii} is the ion–ion collision rate. The form factor h is a function of η_i which satisfies $h(\frac{2}{3}) = 0$ and rises monatonically with η_i to be of order unity at $\eta_i = 2$. The shorter-wavelength modes contribute to the thermal diffusivity through the (collisionless) expression

$$F = 1.4g(\eta_i, L_n/L_s)\tau^{-3/2} \qquad 2 > \eta_i > 0.9 . \tag{2.2.11}$$

A numerical scan of parameter space indicates that, for $L_n/L_s \to 0$, $g(\eta_i) < 0.12$ for $\eta_i \leqslant 2$. Comparing the two transport rates in the region $0.9 < \eta_i < 2$ indicates that the collisional version will dominate if

$$\nu_{ii}\frac{L_s^2}{L_n^2} > \frac{v_{thi}}{L_n} . \tag{2.2.12}$$

Unfortunately no scaling of the form factors is given, merely their maximum values in the range $\eta_i \leqslant 2$ (as quoted above). However, a diagram of numerical values of χ_i as a function of η_i shown in this paper indicates that the collisional version scales like $\sim (\eta_i - \frac{2}{3})^2$ and the collisionless case like $\sim (\eta_i - 0.9)^{1.5}$.

To conclude this subsection on the slab-like modes we mention a 3D simulation of η_i turbulence performed by Kotschenreuther (1991) in which the gyrokinetic approach is compared with the fluid one. No scaling of χ_i is given but it is interesting to note the observed relation

$$\chi_i^{\text{kinetic}} \sim \tfrac{1}{10}\chi_i^{\text{fluid}} \tag{2.2.13}$$

To summarize, early heat diffusivity calculations predicting a rise in χ_i with increasing shear were shown to be in error. More recent theories retain important physics which has been omitted from the early ones to derive scalings of χ_i which *decrease* with shear. This trend is also observed from the higher-order radial eigenmodes. These more recent theories are resonably consistent in their shear scaling, with $\chi_i \sim (L_s/L_n)^2$. Turning to the dependence on η_i there seems to be no one scaling common to all slab theories. All the theories which we have studied here predict an increase of χ_i with η_i but the rate of increase varies: linear for fluid turbulence close to threshold, quadratic for fluid turbulence far above threshold and cubic for gyro-kinetic theory close to threshold. Strictly, fluid theory is not valid close to threshold, in which case the cubic scaling predicted by the gyrokinetic theory is more applicable. Such a strong dependence of the transport on the drive for the turbulence suggests that plasma profiles may be held at marginal stability. Far above threshold, fluid theory is accurate and then we expect $\chi_i \sim \eta_i^2$.

2.3. *Toroidal* ∇T_i *modes*

In recent years there has been an increased effort on transport coefficients due to η_i turbulence with the toroidal effects included and there have been many modifications to the original work of Horton *et al* (1981). We first consider the thresholds for the onset of the turbulence.

Dominguez and Waltz (1988) analyse the linear stability thresholds in the fluid limit for flat density profiles. Although the validity of the fluid limit may be questioned for threshold calculations, this work is important as it demonstrates that stability criteria on η_i become stability criteria on (L_{Ti}/R) in the limit of a flat density profile (as in the case of the slab model discussed in the previous subsection).

In order to calculate the stability threshold one should strictly use kinetic theory as, for example, in the calculation of Biglari *et al* (1989). They consider both peaked and flat density profiles separately. In the case of a peaked profile they apply the ordering

$$\omega_{\rm be}, \omega_{\rm te} \gg \omega_{*\rm i} > \omega_{\rm di} > \omega_{\rm bi}, \omega_{\rm ti} \tag{2.3.1}$$

where $\omega_{\rm bj}$, $\omega_{\rm tj}$ and $\omega_{\rm dj}$ are the species bounce, transit and magnetic drift frequencies, respectively. The first of these constraints ($\omega_{\rm be}, \omega_{\rm te} \gg \omega_{*\rm i}$) is easily satisfied while

$$\omega_{*\rm i} > \omega_{\rm di} \;\Rightarrow\; L_n < \frac{R}{2} \tag{2.3.2}$$

and therefore sets the limitation that the density gradient should not be too flat:

$$\omega_{\rm di} > \omega_{\rm bi}, \omega_{\rm ti} \;\Rightarrow\; k_\theta \rho_{\rm i} > \frac{1}{q}. \tag{2.3.3}$$

This is a severe constraint, particularly close to the plasma centre where q approaches unity. The result of such a constraint is that the ion transit resonance can be neglected, but as shown by Romanelli (1989) this resonance has a small effect. There is, therefore, some justification in ignoring it. The mode frequency, ω is ordered like $\omega_{\rm di}$ so collisions can be neglected if

$$\nu_{\rm i} < \omega_{\rm di} \Rightarrow \nu_{*\rm i} < \epsilon^{-3/2} q k_\theta \rho_{\rm i}\,. \tag{2.3.4}$$

With these constraints the following conditions lead to instability:

$$\eta_{\rm i} < 0 \;\text{ or }\; \eta_{\rm i} > 2/3 \qquad \text{and} \qquad \frac{L_{T\rm i}}{R} < \left(\frac{L_{T\rm i}}{R}\right)_{\rm c} \tag{2.3.5}$$

where $(L_{T\rm i}/R)_{\rm c}$ is some critical value obtained by a numerical solution of the dispersion relation.

The above theory does not apply to a flat density profile because of the condition $\omega_{*i} > \omega_{di}$ which has been applied. In this limit Biglari *et al* demonstrate that the only relevant stability parameter is (L_{Ti}/R). Numerically, for $b_\perp = k_\| L_s = 0$ (where $2b_\perp = (k_\perp \rho_i)^2$) they find

$$\frac{L_{Ti}}{R} \gtrsim 0.35 \tag{2.3.6}$$

for stability in a plasma with a flat density profile. In this case they give the stability diagram sketched in figure 1, valid for all L_n/R. The fluid model of Dominguez and Waltz (1988) (which is in good qualitative agreement with Biglari *et al* 1989) suggests that increasing $b_\perp$ pushes the stability boundary to the left as indicated by the arrow in figure 1.

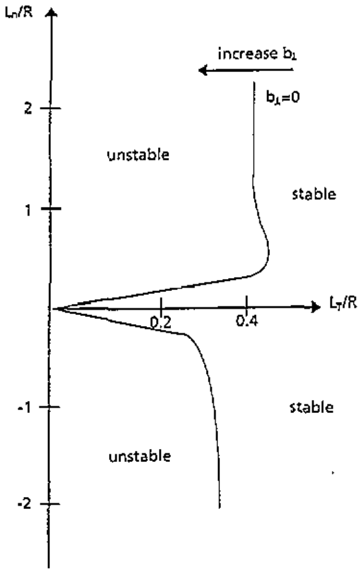

Figure 1. Marginal stability plots for the toroidal ion temperature gradient mode of Biglari *et al* (1989).

Romanelli (1989) derives a threshold for η_i by solving the gyrokinetic drift equation numerically, retaining the ω_d/ω resonance. Using a fluid limit of the equations, Romanelli demonstrates that $\omega \sim \epsilon_n^{1/2}\omega_{*i}$. With this ordering, the neglect of collisions requires

$$\nu_{*i} \ll \frac{q k_\theta \rho_i}{2\epsilon^{3/2}\epsilon_n^{1/2}}. \tag{2.3.7}$$

This is therefore a weaker constraint on the collisionality than that of Biglari *et al* (equation (2.3.4)) if the density scale length is short. Solving the gyrokinetic equation numerically and using a fitting procedure, Romanelli then gives a critical η_i of

$$\eta_{ic} = \begin{cases} 1 & \epsilon_n < 0.2 \\ 1 + 2.5(\epsilon_n - 0.2) & \epsilon_n > 0.2 \end{cases} \tag{2.3.8}$$

as quoted in (2.1.8). For $\epsilon_n \gg 1$ (i.e. the flat density profile case) this result becomes

$$\left(\frac{L_n}{L_{Ti}}\right)_c = 2.5\frac{L_n}{R} \tag{2.3.9}$$

or simply

$$\left(\frac{L_{Ti}}{R}\right)_c = 0.4 \tag{2.3.10}$$

which is similar to that given by Biglari *et al* in (2.3.6).

The critical $\eta_{\rm ic}$ given by Romanelli (1989) in (2.3.8) is independent of both shear and safety factor. Guo and Romanelli (1993) investigate the dependence of the threshold on these using a numerical calculation of the ion gyrokinetic equation in ballooning space. Trapped electron effects are not included and circulating electrons are assumed to be adiabatic. For wavelengths of the order $k_\theta \rho_s \approx \epsilon_T^{1/4}$ they give the following fit to the numerical results:

$$\eta_{\rm ic} = \begin{cases} 1.2 & \epsilon_n < \epsilon_{nc} \\ \frac{4}{3}(1+\tau^{-1})(1+2s/q)\,\epsilon_n & \epsilon_n > \epsilon_{nc} \end{cases} \tag{2.3.11}$$

with

$$\epsilon_{nc} = \frac{0.9}{(1+\tau^{-1})(1+2s/q)}\,. \tag{2.3.12}$$

It is interesting to note that in the slab limit ($s/q \gg 1$) one recovers the scaling of Hahm and Tang shown in (2.2.3). Support for this expression is also provided by the work of Xu and Rosenbluth (1991) with their gyrokinetic particle simulation code. They model the linearized gyrokinetic equation for circular flux surface equilibria in the ballooning limit and investigate the dependence of the threshold on tokamak parameters. Full trapped particle (ion and electron) dynamics are retained. In particular, for $\tau = s = 1$, $q = 2$ they obtain $\epsilon_{T\rm i}^{\rm c} = 0.21$ for $k_\theta \rho_s = 0.4$, while the fit of equation (2.3.11) gives $\epsilon_{T\rm i}^{\rm c} = 0.19$. Xu and Rosenbluth do not distinguish between trapped-ion modes and the circulating ion mode (discussed here). However, the insensitivity of the Xu and Rosenbluth result to the ion collisionality in this parameter regime implies that it is the circulating mode which is studied. A recent numerical treatment by Garbet *et al* (1992) derives a critical temperature gradient $\epsilon_{T\rm i}^{\rm c} \simeq 0.01$ for the circulating mode which is much below the result given here. In fact, the mode is stable for typical tokamak parameters. However, they do find a trapped-ion branch with stability properties very similar to those described by (2.3.11); this mode will be discussed in the next subsection.

The toroidal thresholds discussed so far assume circular flux surfaces. An investigation into the effects of shaping has been made by Hua *et al* (1992) who extend the model of Xu and Rosenbluth (1991) to investigate the effects of triangularity and elongation on ITG mode stability. The temperature gradient threshold is then found to be relatively insensitive to triangularity but has a significant dependence on elongation. In particular, the following expression is derived:

$$\epsilon_{T\rm i}^{\rm c} = \frac{1}{2}\frac{1-M}{1+\tau^{-1}-2M} \tag{2.3.13}$$

where M increases with the elongation, κ, as

$$M = \frac{1}{2q}\sqrt{2s-1+\kappa^2(1-s)^2}\,. \tag{2.3.14}$$

In the limit $\kappa \to 1$ this expression depends on τ and s/q and, although the magnitude is similar to that obtained by Guo and Romanelli (1993), the scaling is slightly different.

Trapped-electron dynamics have an interesting role to play in the stability analysis and can give a significant modification to figure 1 (Romanelli and Briguglio 1990; Nordman *et al* 1990; Guo and Romanelli 1993). For flat density profiles stability is not affected significantly. However, for low ϵ_n values, the trapped-electron dynamics destabilize the mode and, as illustrated schematically in figure 2, no stable window exists for peaked density profiles. Increasing electron collisionality ($\nu_{*e} \gtrsim 1$) suppresses trapped-electron effects and the stability diagram returns to that shown in figure 1.

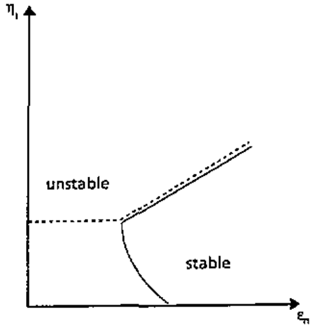

Figure 2. Marginal-stability curves in the η_i–ϵ_n plane for the ITG mode including trapped-electron effects. The full curve is for $\nu_{*e} = 0$ and the broken curve neglects trapped electrons.

This completes our survey of the thresholds and we turn now to recent predictions of the ion heat transport. Biglari *et al* (1989) derive an expression for the thermal diffusivity, χ_i, using the fluid equations and employing mixing-length estimates. The fluid approximation requires $\eta_i \gg 1$ and then

$$F = (k_\theta \rho_s)\left(\frac{q}{s}\right)\frac{(1+\eta_i)}{\tau}. \tag{2.3.15}$$

An upper bound on $k_\theta \rho_i$ of $(1+\eta_i)^{-1/2}$ (as obtained from a linear analysis) can be used to obtain a simple expression for the upper limit of χ_i. A gyrokinetic simulation of the fluctuation spectrum by Sydora *et al* (1990) (and supported by the more recent simulation of Xu and Rosenbluth 1991) shows that the important range of $k_\theta \rho_i$ is $0.1 < k_\theta \rho_i < 0.5$, suggesting that this upper bound on χ_i could be close to the actual value.

It is interesting to note that Biglari *et al* also give a scaling for the electron thermal diffusivity, χ_e, which results from the electron transport caused by ion-pressure-gradient-driven turbulence:

$$\chi_e \sim \epsilon^{3/2}(k_\theta \rho_s)^2 \frac{c_s^2 \rho_s^2}{L_n^{3/2} R^{1/2} \nu_e}\left(\frac{1+\eta_i}{\tau}\right)^{3/2}\frac{q}{s}\left(\frac{1}{1+0.1/\nu_{*e}}\right). \tag{2.3.16}$$

The last factor on the right is a correction factor which has been added to the Biglari result to allow it to be extended to a low collisionality regime, representing the transition from dissipation due to collisional effects to dissipation due to the magnetic drift resonance (Romanelli *et al* 1986). They also give the particle diffusion coefficient, D

$$D \sim \chi_e \tag{2.3.17}$$

where they take the non-adiabatic part of the electron response to the ITG driven potential fluctuations to be due to dissipative trapped-electron dynamics.

Guo *et al* (1989) use the fluid limit of the electrostatic gyrokinetic equations in their analysis and consider flat density profiles so that $\eta_i \gg \eta_c$. χ_i is calculated from the estimate $\gamma/k_\perp^2$ with $\epsilon_T \ll 1$ assumed in calculating γ. Expressions for χ_i are then calculated in two different wavelength regimes (a) the weak ballooning limit ($b_\theta \ll \epsilon_T^{1/2}$) and (b) the strong ballooning limit ($1 \gg b_\theta \gg \epsilon_T^{1/2}$). A maximum value of χ_i occurs for the b_θ for which these match leading to

$$F = 0.6\left(\frac{L_n}{R}\right)\left[\frac{(1+2q^2)^3}{sq(\tau\epsilon_T)^3}\right]^{1/4}. \tag{2.3.18}$$

Strictly, q here is the cylindrical q_c but for large aspect ratio there is no need to distinguish between the two.

Hong and Horton (1990) solve the 2D fluid equations describing the toroidal η_i instability and use mixing-length estimates to derive the ion thermal diffusivity. The fact that fluid equations have been used implies that the results can only be applied when η_i is far above threshold. For $\rho_s/L_n < s < 2\epsilon_n$ they derive

$$F = 2\frac{L_n}{R}\frac{1}{\tau^{1/2}s}(\eta_i - 2/3)^{1/2} \tag{2.3.19}$$

and for very low shear ($s < \rho_s/L_n$) the radial mode width $\Delta x \sim L_n$ thus giving a Bohm scaling for χ_i

$$\chi_i = \tau^{3/4}\epsilon_n^{3/4}(\eta_i - \eta_c)^{1/4}\rho_s c_s . \tag{2.3.20}$$

Using a modified mixing-length approximation (Horton *et al* 1981), Dominguez and Waltz (1989) derive the following scaling for F

$$F = 3.53\left(\frac{c_i}{k_\perp \rho_s}\right)\frac{q}{s}\frac{L_n}{L_{T_i}}\sqrt{1 - \frac{L_{T_i}}{L_{T_i}^c}}\,\Theta\left(1 - \frac{L_{T_i}}{L_{T_i}^c}\right) \tag{2.3.21}$$

where $\Theta(x)$ is the Heaviside function and

$$\frac{1}{L_{T_i}^c} = \left[\frac{c_1}{R}, \frac{c_2}{L_n}\right]_{\max} \tag{2.3.22}$$

with $c_2 \sim 1$ and c_1 in the range

$$0.2 \leqslant \frac{L_{T_i}^c}{R} \leqslant 0.3 \tag{2.3.23}$$

The numerical coefficient, c_i, is taken to be 0.3 and $k_\perp \rho_s \sim 0.3$. Again, the result is only applicable to plasmas in which $\eta_i \gg 1$.

The earlier work of Horton *et al* (1981) has since been extended by Hong *et al* (1986) to include kinetic effects. They use gyrokinetic theory for the ions and adiabatic electrons, and consider the frequency range $\omega \gg \omega_{di} > k_\parallel v_{\parallel i}$, which imply

$$\epsilon_n \ll \left(\frac{\tau}{2}\right)^{1/2} \qquad \text{and} \qquad \left(\frac{s}{q}\right)^{1/2}\left(\frac{1+\eta_i}{\epsilon_n}\right)^{1/4} \ll 1 \tag{2.3.24}$$

i.e. the density profile must not be too flat and the shear must not be too large. With these restrictions and considering $\eta_i \gtrsim 2$, Hong *et al* derive the following scaling for F:

$$F = 2\frac{q}{s}\left(\frac{1+\eta_i}{\tau}\right)k_0\left[1 - \left(\frac{1+\eta_i}{2\gamma_0\tau}\right)^2 k_0^4\right]^{-1} \tag{2.3.25}$$

where

$$\gamma_0^2 = 2\epsilon_n\frac{(1+\eta_i)}{\tau} \tag{2.3.26}$$

$$k_0 = \left[\frac{2(1-2\epsilon_n) + \{(1-2\epsilon_n)^2 + 24\epsilon_n[(1+\eta_i)/\tau]\}^{1/2}}{3(1+\eta_i)/\tau}\right]^{1/2} . \tag{2.3.27}$$

Romanelli (1989) uses a kinetic ion response without expanding in ω_d/ω and employs a quasilinear mixing-length estimate to derive an expression for χ_i. Because kinetic effects are kept, the theory is valid down to very low η_i, close to the threshold value. The calculation of the ion energy flux assumes

$$\epsilon_T \ll 1 \qquad \text{and} \qquad \eta_i > 2\epsilon_n \tag{2.3.28}$$

together with condition (2.3.7). Taking $k_\theta^2 \rho_s^2 = 0.1\tau^{-1}$, Romanelli obtains

$$F = 14\frac{\epsilon_n^{1/2}}{\tau^{3/2}}(\eta_i - \eta_{ic})^{1/2} \tag{2.3.29}$$

with η_{ic} given in (2.3.8). In a later work Romanelli and Briguglio (1990) use kinetic theory to investigate the micro-instabilities that are driven by trapped electrons and ion temperature gradients. The expressions for the fluxes are rather complicated and we refer the interested reader to appendix C of the reference.

A recent article by Romanelli *et al* (1991) describes a kinetic theory of the ion temperature gradient driven mode in the limit of long wavelength, $k_\theta \rho_i \sim \epsilon_T$. They perform a calculation in toroidal geometry and find that three modes are important at these wavelengths: a slab-like mode and two toroidal modes. The most important toroidal mode propagates in the ion diamagnetic drift direction (and will be referred to as the ion toroidal mode). This mode is shown to be unstable whenever a parameter λ,

$$\lambda = \frac{q k_\theta \rho_i}{\sqrt{2}\epsilon_T} \tag{2.3.30}$$

exceeds a critical value, λ_c, where

$$\lambda_c = \frac{1}{2}\left(\frac{2}{\tau}\frac{1+\tau^{-1}}{1-(2/\eta_i)}\right)^{1/2} . \tag{2.3.31}$$

This can never be satisfied if $\eta_i < 2$ and therefore the toroidal ion mode is stable for all wavelengths in this case. For values of $\eta_i > 2$ the modes with wavelength short enough such that $\lambda > \lambda_c$ will be unstable. There can be significant transport from this ion toroidal mode and in the fluid limit

$$\lambda \gg 1 \tag{2.3.32}$$

the following thermal diffusivity is given:

$$F \simeq \frac{1}{\sqrt{2}\tau^{3/2}}\frac{q\eta_i}{s^2} . \tag{2.3.33}$$

The second toroidal mode propagates in the electron diamagnetic direction and is therefore referred to as the electron toroidal mode. This is found to be marginally stable in the absence of kinetic effects. Inclusion of a trapped-electron response can drive this mode unstable, though it is less important for transport than the ion mode and no expression for χ_i is given. Trapped electrons can have a major influence on the stability of this mode for tight aspect ratio ($\epsilon \gtrsim 0.3$) and low collisionality $\nu_{*e} \lesssim 1$.

The two modes described so far are peculiar to toroidal geometry and there is no analogue for them in a slab geometry. The third mode which is found does have a slab-like analogue and is therefore termed the slab mode. This is considered in the long-wavelength limit, with the ordering $k_\theta \rho_i \lesssim \epsilon_T^{1/2}$ (i.e. shorter wavelengths than were allowed for the toroidal calculation). The growth rate is derived far above threshold allowing the following mixing-length estimate of the thermal diffusivity to be made:

$$F \simeq 0.5\frac{q}{\tau}\left(\frac{\eta_i \epsilon_n}{s}\right)^{1/2} . \tag{2.3.34}$$

The assumptions made in deriving this are

$$7\pi^{1/2}(\epsilon_T s\tau)^2 \ll \lambda \ll [7\pi^{1/2}(\epsilon_T s\tau)^2]^{-1/4} \tag{2.3.35}$$

where the upper bound has been used to estimate k_θ in calculating χ_i, and that the density profile should be sufficiently flat

$$\eta_i \gg \left(\frac{32}{7\pi^{1/2}}\right)\frac{\tau^{3/5}\lambda^{1/5}}{(\epsilon_T s)^{2/5}}. \qquad (2.3.36)$$

Nordman *et al* (1990) adopt a fluid approach (claimed to be in close agreement with kinetic theory) to analyse the η_i mode, taking into account non-adiabatic trapped electron dynamics. The resulting quartic dispersion relation has two branches which can be unstable simultaneously (as discussed earlier)—a collisionless trapped-electron mode and a toroidal η_i mode enhanced by trapped-electron dynamics (see figure 2). A modified saturation level is derived by balancing the linear growth ($\sim \gamma\delta n$) against the convective nonlinearity ($\sim \boldsymbol{v}_E.\nabla\delta n$, where $\boldsymbol{v}_E$ is the $\boldsymbol{E}\times\boldsymbol{B}$ drift) leading to

$$\frac{e\phi}{T_e} = \frac{\gamma}{\omega_{*e}}\frac{1}{k_x L_n}. \qquad (2.3.37)$$

Using this in a quasilinear calculation of the ion heat flux then yields

$$\chi_i = \frac{1}{\eta_i}\left[\eta_i - \left\{\frac{2}{3} + (1-f_t)\frac{10}{9\tau}\epsilon_n + \frac{2}{3}f_t\Delta_i\right\}\right]\frac{\gamma^3/k_x^2}{(\omega_r + 5/3\omega_{di})^2 + \gamma^2} \qquad (2.3.38)$$

where f_t is the trapped particle fraction and Δ_i is a function of the mode frequency ω_r and growth rate γ given in (16) of Nordman *et al* (1990). Determination of γ and ω_r from the dispersion relation then yields an ion thermal diffusivity which compares well with their full numerical simulation. It is interesting to note that predicted radial profiles of χ_i and χ_e for a typical JET shot rise towards the plasma edge out to $r/a \sim 0.9$ in qualitative agreement with experimental trends. Over the outer 10% of the plasma, however, both χ_i and χ_e fall sharply towards zero in contradiction with experimental measurements. Another interesting feature of this theory is the heat pinch (given by the term in curly brackets of (2.3.38)) which is proportional to the density gradient. A later work (Weiland and Nordman 1991) based on the same model applied near the tokamak edge (where $\epsilon_n \ll 1$) produces similar χ profiles. Other features of this $\epsilon_n \ll 1$ limit are:

(1) stabilization of the most dangerous mode (accompanied by a corresponding decrease in χ) when a power threshold is exceeded (this is interpreted as a possible L–H transition mechanism) and
(2) stabilization of the most dangerous mode at tight aspect ratio.

Finally we discuss the work of Kim *et al* (1991) who use neoclassical fluid equations. The frequency ordering that they impose (i.e. $\omega \sim \omega_{*i} \ll \omega_{bi}$) leads to the restriction

$$\frac{\tau^{1/2}\epsilon^{1/2}\epsilon_n}{q} \gg \frac{k_\theta\rho_s}{\sqrt{2}} \sim 0.2. \qquad (2.3.39)$$

The ion collisionality, ν_{*i}, is limited to $\nu_{*i} \ll \epsilon^{-3/2}$ because of the restriction that the static neoclassical viscous damping frequency

$$\mu_i = \frac{0.78\epsilon^{1/2}\nu_{ii}}{1 + 0.44\nu_*} \qquad (2.3.40)$$

satisfies $\mu_i \ll \omega_{bi}$. The fluid approach requires that η_i must be far above threshold; an upper bound on χ_i is given as

$$\chi_i < 2.3\mu_i\rho_s^2\frac{q^2}{\epsilon^2}(1+\eta_i) \qquad (2.3.41)$$

i.e. they find that the transport is enhanced above neoclassical by a factor of the order $(1+\eta_i)$.

2.4. Trapped-ion modes

We first discuss the threshold conditions for the onset of these modes. The stability of the ITG family of instabilities including trapped-ion modes, has been investigated by Garbet *et al* (1992) who derive a dispersion relation which describes three modes. Two of these are associated with the circulating particles and correspond to the η_i mode and an interchange-type mode; the third is a trapped-ion mode. The dispersion relation for this mode is solved numerically in the collisionless limit and within the framework of the ballooning formalism. Curves of marginal stability are then drawn in the L_n–$L_{T\mathrm{i}}$ plane. A sketch of their results for the trapped-ion modes is illustrated in figure 3 for two values of the toroidal mode number, n, for a typical JET shot (i.e. $q_0 = 1.5$, $T_\mathrm{e} = T_\mathrm{i} = 1750$ eV, $B_0 = 2.75$ T and $\mathrm{d}q/\mathrm{d}\psi = 0.7\ \mathrm{Wb}^{-1}$). For low positive η_i they observe that there exists a critical η_i, (e.g. $\eta_{\mathrm{ic}} = \frac{2}{3}$ for $n = 30$ or $\eta_{\mathrm{ic}} = 1$ for $n = 150$), below which the trapped ion mode is stable. For higher η_i they observe that the stability criterion becomes a threshold on the ion temperature length scale, $L_{T\mathrm{i}}$, rather than on η_i. This corresponds to $L_{T\mathrm{i}}/R = 0.17$ for $n = 30$ and $L_{T\mathrm{i}}/R = 0.1$ for $n = 150$. This plot has similar characteristics to the conventional η_i mode and thus comparisons with this trapped-ion mode are useful. Garbet *et al* calculate the critical $L_{T\mathrm{i}}/R$ for the η_i mode of the circulating particles using the same parameters as were used for the trapped-ion mode. They find a value of $(L_{T\mathrm{i}}/R)_\mathrm{c} \sim 0.01$ and conclude that the trapped-ion mode is more important in tokamaks than the η_i mode (this critical temperature gradient is very much below those which were described in section 2.3). The critical value of the temperature length scale, below which trapped-ion modes are unstable, is dependent upon the toroidal mode number, n, with lower values of n having higher thresholds.

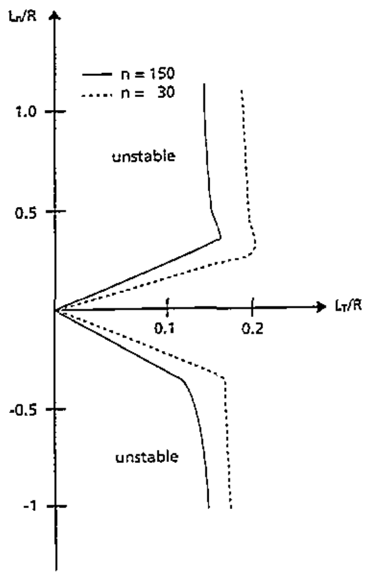

Figure 3. Marginal-stability plots for the trapped-ion mode of Garbet *et al* (1992).

The threshold for instability to the trapped-ion modes has also been calculated for the collisionless mode by Dominguez (1990). As well as the collisionless assumption, the calculation is performed in the limit of large aspect ratio, requiring

$$\nu_* \ll 1 \qquad \epsilon \ll 1\,. \tag{2.4.1}$$

The form of the threshold for instability is

$$\epsilon_n < f(\eta_e) \tag{2.4.2}$$

where $f(\eta_e)$ is a complicated function which can be found in the reference. By controlling η_e Dominguez claims that it is possible to move the threshold and help stabilize the trapped-ion mode.

We now turn to the various trapped-ion transport models that have been proposed recently. We begin with the paper of Diamond and Biglari (1990) in which earlier work on dissipative trapped-ion convective cell turbulence driven by electron collisions (Cohen *et al* 1976) is reconsidered. In the work of Cohen *et al*, the two-dimensional $\boldsymbol{E} \times \boldsymbol{B}$ advective nonlinearity was dropped and a one-dimensional nonlinear 'shock' term retained. This model led to a diffusion coefficient, D, which scaled as $D \sim T^{21/2}$, corresponding to very large anomalous transport, particularly at high temperatures. However, when the temperature is high the $\boldsymbol{E} \times \boldsymbol{B}$ term is the dominant nonlinearity and the calculation of Cohen *et al* becomes invalid. Such a two-dimensional mechanism is better able to transfer the unstable fluctuation energy to where it can be dissipated than the one-dimensional version and leads to a steady state with less transport. The condition that the $\boldsymbol{E} \times \boldsymbol{B}$ nonlinearity dominates over the shock nonlinearity is

$$\nu_{*e} \ll \sqrt{2} s k_\theta^2 \left(\frac{m_e}{m_i}\right)^{1/2} \frac{\rho_s \eta_i R q}{\epsilon^{1/2}} \tag{2.4.3}$$

where equal temperatures of the ions and electrons have been assumed. Diamond and Biglari then find that

$$F = \frac{3}{2\sqrt{2}} \frac{\epsilon^{1/2}}{s^2} \left(\frac{m_e}{m_i}\right)^{1/2} \frac{q}{\epsilon_n \nu_{*e}} . \tag{2.4.4}$$

Expressions for the electron thermal (χ_e) and particle (D) diffusivities are also given in terms of χ_i as

$$\chi_e \simeq \tfrac{3}{2} D \simeq \chi_i . \tag{2.4.5}$$

The trapped-ion temperature-gradient-driven mode is considered using a two-point renormalized (clump) theory by Biglari *et al* (1988). In clump theory the fact that the nonlinear interaction of modes can produce fluctuations which are not in phase with the potential (i.e. incoherent fluctuations) is considered. The word 'clump' describes the phase-space granulation resulting from turbulent mixing (i.e. 'clumps' of plasma are formed which, to a certain extent, behave as a single large particle). The mode, which is shown to propagate in the ion drift direction, is driven by unfavourable magnetic curvature, unlike the 'conventional' slab η_i mode which is driven by a sound wave and propagates in the electron drift direction. Whilst the ions are treated using clump theory, the electrons are assumed to be sufficiently collisional that electron clumps are not formed. This imposes the constraint on the electron collisionality, ν_{*e}, that

$$\nu_{*e} \gg \frac{(k_\theta \rho_s) q \eta_i}{\sqrt{2} \epsilon^{1/4} s \epsilon_n \tau} \left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{\eta_i}{\eta_{ic}} - 1\right)^{1/2} \tag{2.4.6}$$

where η_{ic} is the critical η_i for the onset of the instability and is given by

$$\eta_{ic} = \left(\tfrac{3}{2} - \bar{E}\right)^{-1} \tag{2.4.7}$$

with $\bar{E}$ the ratio of the ion kinetic energy to the ion thermal energy. Other frequency orderings impose the following constraints:

$$\nu_{*i}, \nu_{*e} \ll 1 \qquad \omega_{bi}, \omega_{ti} \gg \omega_{*i} \Rightarrow k_\theta \rho_s \ll \frac{(2\epsilon\tau)^{1/2} \epsilon_n}{q} \qquad \omega_{*i} \gg \omega_{di} \Rightarrow \epsilon_n \ll 1 . \tag{2.4.8}$$

These imply that the results are valid for long-wavelength modes in low collisionality plasmas for which the density profile is not too flat. The following expressions for the ion particle (Γ_i) and thermal (Q_i) fluxes are given:

$$\begin{pmatrix} \Gamma_i \\ Q_i \end{pmatrix} \sim -\epsilon^{3/2}\left(\frac{c_s\rho_s^2}{L_n}\right)\left(\frac{c_s}{L_n\nu_e}\right)\left(\frac{\eta_i}{s\tau}\right)^2 \times\left(\frac{\eta_i}{\eta_{ic}\mid\omega/\omega_{di}\mid}-1\right)n_i\begin{pmatrix} \mathrm{d}\ln n_i/\mathrm{d}r \\ \mid\omega/\omega_{di}\mid \mathrm{d}T_i/\mathrm{d}r \end{pmatrix} \qquad (2.4.9)$$

where $\mid\omega/\omega_{di}\mid$ is constrained to lie in the range

$$\tfrac{3}{2} > \left|\frac{\omega}{\omega_{di}}\right| > 1\,. \qquad (2.4.10)$$

The lower bound is necessary to achieve nonlinear saturation, and the upper bound to have nonlinear instability. If we use the lower value then we obtain the following expression for F:

$$F \simeq \frac{1}{\sqrt{2}}\left(\frac{m_e}{m_i}\right)^{1/2}\frac{q}{\epsilon_n\nu_{*e}}\left(\frac{\eta_i}{s\tau}\right)^2\left(\frac{\eta_i}{\eta_{ic}}-1\right). \qquad (2.4.11)$$

The work of Biglari *et al* (1989), which we considered earlier in relation to the toroidal η_i mode, also calculates transport due to the trapped ions. Trapped-ion pressure-gradient-driven modes are considered both in a collisionless model and a model where a pitch-angle scattering collision operator is included. The frequencies are ordered according to

$$\omega_{bi},\ \omega_{ti} \gg \omega_{*i} > \mid\omega\mid \gtrsim \omega_{di},\ \nu_{eff,i}\,. \qquad (2.4.12)$$

These imply that the plasma should be of low collisionality:

$$\nu_{*i} \ll 1 \qquad (2.4.13)$$

the density profile should not be too flat:

$$\epsilon_n \ll \tfrac{1}{2} \qquad (2.4.14)$$

and the modes are of long wavelength:

$$k_\theta\rho_s \ll \frac{(2\tau\epsilon)^{1/2}\epsilon_n}{q}\,. \qquad (2.4.15)$$

For the collisionless case, a criterion for instability is derived:

$$\frac{2(1+\tau)}{\sqrt{2\epsilon}}\frac{(1+\eta_i)}{\tau^2} > \frac{1}{2\epsilon_n} \qquad (2.4.16)$$

which, when satisfied, leads to the following mixing-length result for the ion thermal diffusivity:

$$F \sim 2^{3/4}\frac{\epsilon^{1/4}\epsilon_n^{1/2}}{(k_\theta\rho_s)s^2}\left(1+\frac{\eta_i}{\tau}\right)^{1/2}. \qquad (2.4.17)$$

The effects of perturbatively including ion collisions via a pitch-angle scattering operator are then studied. The dissipative trapped-ion mode was found to be stabilized by ion collisions when $\eta_i = 0$ in an early work by Kadomtsev and Pogutse (1971). Here, the finite η_i regime is explored and it is found that when η_i exceeds a critical value ($\eta_{ic} = \frac{4}{3}$) ion collisions actually have a destabilizing influence. Thus when

$$\eta_i > \tfrac{4}{3} \qquad \nu_{*i} < q(k_\theta\rho_s)\left(\frac{\tau^2}{\epsilon_n^2\epsilon}\right)^{1/4} \qquad (2.4.18)$$

the mode is unstable and the thermal diffusivity corresponds to

$$F = 2\sqrt{2}\frac{\eta_i q \tau^{1/2}}{s^2 \epsilon^{1/4} \nu_{*i}}. \tag{2.4.19}$$

This value for the thermal diffusivity is rather high and has a large unfavourable temperature scaling; the instability is therefore expected to fix the plasma profiles at a level corresponding to marginal stability. This value for F is also somewhat larger than that obtained by Diamond and Biglari (1990) for the dissipative trapped-ion mode. In that case $\eta_i = 0$ and the electron collisions were the main driving mechanism, whereas here η_i is assumed to exceed the threshold (of $\frac{4}{3}$) and the ion collisions also become destabilizing.

A similar result to (2.4.17) is obtained by Biglari and Diamond (1991) when the mode is analysed with a fluid frequency ordering ($\omega_{bi} \gg |\omega| \gg \omega_{di} > \nu_{effi}$). Assuming equal ion and electron temperatures they derive a fluid model describing the evolution of the density and temperature fluctuations. The saturated levels of density and temperature fluctuations are calculated from which an ion heat diffusivity is derived, corresponding to

$$F = 2\frac{(\epsilon\epsilon_n\eta_i)^{1/2}}{s^2}. \tag{2.4.20}$$

Expressions for the particle and electron heat diffusion are also given:

$$D \sim \chi_e \sim 4\frac{\epsilon^{1/2}}{s^2}\frac{\rho_s^2 c_s^2}{L_n^2 \nu_{\mathrm{eff,e}}}\eta_i\epsilon_n. \tag{2.4.21}$$

Xu and Rosenbluth (1990) have considered the stability criterion for the trapped-ion mode and its relation to certain other instabilities. For low-frequency, long-wavelength modes a general analytic dispersion relation is derived which contains three types of instability—electrostatic trapped-ion modes (i.e. trapped-ion modes that are electrostatic in nature), magnetic trapped-ion modes (i.e. trapped-ion modes which have no electrostatic contribution) and MHD ballooning instabilities. The dispersion relation is obtained using a variational approach constructed from gyrokinetic equations. It is found that the couplings between the modes are weak (except for the case of the ballooning mode, where it is found that the trapped particles are stabilizing). By considering different mode frequency orderings they separate out the trapped-ion modes and evaluate thermal diffusivities and stability criteria for different collisionality regimes. The effects of collisions are incorporated into the model via a pitch-angle scattering operator but the effects of trapped electrons are not considered. We concentrate here on the results which they obtain for the trapped-ion modes, beginning with the electrostatic mode which is considered in the region $\frac{4}{3} < \eta_i < \epsilon^{-1/2}$ when ion collisions are destabilizing. Thermal diffusivities for two collisionality regimes—'collisionless' and 'collisional' are derived. For the collisionless case, the ion collision frequency must be less than the drift frequency:

$$\nu_{*i} \ll a_1 \frac{\sqrt{2}q(k_\theta\rho_s)}{\epsilon^{1/2}\tau^{1/2}} \tag{2.4.22}$$

where

$$a_1 = 0.36 + 0.21s - 0.21\left(1 + \frac{7}{6q^2}\right)\alpha \qquad \alpha = -q^2 R\frac{\mathrm{d}\beta}{\mathrm{d}r} \tag{2.4.23}$$

and β is the ratio of thermal to magnetic energy. The frequency ordering which is imposed gives the following constraints on the mode wavelength:

$$\frac{\tau^{1/2}\epsilon^{1/2}\nu_{*i}}{\sqrt{2}q} \ll k_\theta\rho_s \ll \frac{\tau^{1/2}\epsilon^{3/4}}{\sqrt{2}q}. \tag{2.4.24}$$

It is shown that a mode driven by ion pressure gradient is unstable for α below a critical value, α_c, where

$$\alpha_c = \frac{1.71 + s}{1 + 7/(6q^2)} \tag{2.4.25}$$

and the thermal diffusivity in this situation corresponds to

$$F = \frac{2\sqrt{2}\epsilon^{3/4}\epsilon_n^{1/2} a_1^{3/2}}{[1 + h^2(\theta)]\tau^{3/2}} \frac{\eta_i^{1/2} q}{\nu_{*i}} \tag{2.4.26}$$

where

$$h(\theta) = s\theta - \alpha \sin\theta \tag{2.4.27}$$

and θ is the poloidal angle.

For the collisional case the ion collision frequency exceeds the drift frequency:

$$\nu_{*i} \gg a_1 \frac{\sqrt{2}q(k_\theta \rho_s)}{\epsilon^{1/2}\tau^{1/2}} \tag{2.4.28}$$

and then the frequency ordering which is imposed leads to the following bound on the mode wavelength:

$$k_\theta \rho_s \gg \frac{\epsilon_n \nu_{*i} \tau^{1/2}}{\sqrt{2}\eta_i q} . \tag{2.4.29}$$

The corresponding thermal diffusivity is given by

$$F = 2\sqrt{2}\left(\frac{\epsilon}{\tau}\right)^{3/2} \frac{1}{[1 + h^2(\theta)]} \frac{q\eta_i^2}{\nu_{*i}\epsilon_n} . \tag{2.4.30}$$

Although there are similarities, the ϵ dependence, for example, is quite different from that in the collisional result of Biglari *et al* (1989) (see (2.4.19)).

Finally, we turn to the purely magnetic mode where the frequency ordering requires

$$k_\theta \rho_s \gg \sqrt{2}\frac{\epsilon_n \epsilon^{1/2} \nu_{*i} \tau^{1/2}}{q} . \tag{2.4.31}$$

The condition that the mode be unstable can be expressed as a condition on the collisionality:

$$\nu_{*i} < 5.7 a_1^{1/2} [\beta(1 + 3\eta_i)]^{1/2} \frac{\epsilon^{-1/4}(k_\theta \rho_s) q}{\tau^{1/2}\epsilon_n^{1/2}} \tag{2.4.32}$$

and the resulting thermal diffusivity can be calculated from

$$F \sim 2\sqrt{2} a_1 \frac{q\beta}{\nu_{*i}\tau^{3/2}[1 + h^2(\theta)]}(1 + 3\eta_i) . \tag{2.4.33}$$

2.5. *Conclusions*

There is a strong belief that toroidal ∇T_i modes play a role in tokamak transport and much effort has been devoted to refining the onset conditions and consequent transport to improve agreement with experimental results on χ_i. Although the scalings for these theoretical gyro-Bohm expressions for χ_i have improved through the geometric factors appearing in the functions F, problems remain. Recent comparisons with JET experimental data made by Connor *et al* (1993) indicate that all the above ITG turbulence-driven transport theories tend to have a common problem—a failure to reproduce the rise in the ion thermal diffusivity observed towards the tokamak edge. On the contrary, as a consequence of the strong temperature dependence associated with their gyro-Bohm scaling all theories here predict a *fall* in χ_i towards the plasma edge. Current developments in theory attempt to address this problem in different ways. Hua *et al* (1992) explain the radial profile by retaining threshold effects whereas the majority of the theories quoted above are strictly valid far above threshold. Hua *et al* postulate that in the core of the plasma the density and temperature are held close to marginally stable profiles and little transport results in this region (typically much less than predicted by the above models). Towards the edge, boundary conditions (such as the density and temperature tending towards zero) force the plasma profiles away from marginal stability and so the diffusivity rises to the values typical of those quoted above. Beklemishev and Horton (1992) claim that one should weight the heat diffusivity by a 'density of states' factor which reflects how closely the mode rational surfaces are packed. This leads to an enhancement of the thermal diffusivity at the tokamak edge which may explain the observed increase in χ_i in this region. Finally Romanelli and Zonca (1993) examine the consequences of toroidal coupling on the radial structure of ITG modes, finding a more general class of instability than the 'ballooning' type modes which we have been discussing. These new modes are similar to those discussed by Connor *et al* (1993) in that they have a much greater radial extent and may therefore be more important for transport. No detailed transport calculations have been made but qualitative comments can be made from the radial structure which Romanelli and Zonca derive. In particular for low shear values ($s < 1$) towards the plasma core, the toroidal coupling of the Fourier modes is very weak ($\sim e^{-1/s}$) and the radial mode width, Δx, is then governed by the width of the individual Fourier modes. Thus for $s < \rho_s/L_n$, as discussed by Hong and Horton (1990), $\Delta x \sim L_n$ and the transport is Bohm-like (see (2.3.20)). For higher shear values in the range $\rho_s/L_n \lesssim s < \epsilon_n$, $\Delta x \sim \rho_s$ and the conventional gyro-Bohm scaling observed in the majority of the theories discussed in this section is obtained. For $s \gtrsim 1$ Romanelli and Zonca (1993) demonstrate strong toroidal coupling of the individual Fourier modes which again gives radially extended structures with $\Delta x \sim a/s^{1/2}$, where a denotes an equilibrium scale length. Such large structures would be expected to give high Bohm-like transport and may, therefore, provide an interpretation of the observed increase in χ_i at the tokamak plasma edge.

3. Electron transport due to drift-wave turbulence

3.1. Overview

Electrostatic drift-wave turbulence can be excited by various mechanisms. The basic drift wave has $\omega \sim \omega_{*e}$ and $k_\perp \rho_s \sim O(1)$. Destabilizing mechanisms are provided by collisions, Landau resonances and trapped particle effects, which must offset shear damping for instability. Shorter-wavelength electromagnetic fluctuations with $k_\perp c/\omega_{pe} \lesssim 1$ can be excited by the η_e (or electron-temperature gradient) mode. The presence of such electromagnetic drift-wave turbulent spectra can produce stochastic transport of test particles, particularly trapped ones, which is largely independent of the origin of the turbulence—this is discussed in section 3.2.

In section 3.3 we consider electrostatic electron drift-wave turbulence and transport in more detail. We concentrate on placing the many contributions on this topic in context, emphasizing developments over the last fifteen years. As in the case of the ∇T_i mode, the simplest description of the electron drift wave is in a slab geometry, where it is found that to overcome the damping effect of the magnetic shear it is necessary to introduce a nonlinear theory; such a mode is discussed in section 3.3.1. We then consider the effects of introducing toroidal geometry in 3.3.2; as is well known, toroidicity tends to reduce the shear damping and thus drive the mode more unstable. Toroidal geometry also introduces trapped particles and in 3.3.3 we consider the instabilities and transport that are induced by a trapped electron population. These modes have been studied extensively in the literature in a variety of regimes, using different mode structures and saturation mechanisms and we categorize the calculations according to those. We then consider the effects of inverted density profiles and finally a short-wavelength trapped-electron mode. Section 3.4 discusses the electromagnetic drift wave, the drift micro-tearing modes and the η_e mode in both slab and toroidal geometry.

All the transport coefficients in these subsections are of the gyro-Bohm-type, involving ρ_s or c/ω_{pe} (or both) as microscopic lengths as discussed in section 3.5

3.2. General electromagnetic fluctuations

A turbulent plasma in which the electromagnetic fluctuations are sufficiently large can produce stochastic transport of particles. Three papers by Horton (1985), Parail and Yushmanov (1985) and Horton *et al* (1987) derive the requirements for stochastic motion and the implications such a motion has for transport levels. They numerically solve the equations of motion of the particles under the influence of a model spectrum of electromagnetic fluctuations and calculate a diffusion coefficient, D. The source of the electromagnetic fluctuations is not specified although it is assumed that they could originate from drift waves. The dependence of D on the fluctuation spectrum parameters (e.g. fluctuation amplitude, power-law index of the wavenumber spectrum, mean phase velocity of the fluctuations, etc.) is then studied extensively. It is found that D is relatively insensitive to these properties of the fluctuation spectrum.

In a random-walk estimate of the diffusion coefficient for a highly turbulent plasma of the type discussed above, the relevant decorrelation frequency is the circulation (or $\boldsymbol{E}\times\boldsymbol{B}$ trapping) frequency, Ω_E, defined as

$$\Omega_E = \frac{ck_\perp^2\varphi_k}{B} \tag{3.2.1}$$

where φ_k is a Fourier component of the potential corresponding to wavenumber k and the characteristic step length is $k_\perp^{-1}$. Because trapped particles are localized along a magnetic field-line they do not experience the whole variation of the perturbation along $\boldsymbol{B}$. As a result the fluctuations influence the trapped-particle orbits more than the passing-particle orbits (which will tend to average out the effects of the fluctuations). Thus the dominant contribution to the transport is from the trapped electrons so that

$$\chi_e \sim \epsilon^{1/2}\frac{\Omega_E}{k_\perp^2}\,. \tag{3.2.2}$$

In his analysis, Horton (1985) showed that a condition for such stochastic motion is

$$\Omega_E \sim \Delta\omega$$

where $\Delta\omega$ is the dispersion of the distribution of frequencies of the driving instability. For electron drift waves

$$\Delta\omega \sim \omega_{*e} = k_\perp\rho_s\frac{c_s}{L_n} \tag{3.2.3}$$

and the wavelength is estimated by

$$k_\perp\rho_s \sim 1\,. \tag{3.2.4}$$

Thus the electron heat diffusivity is given by

$$\chi_e^{(1)} \sim \epsilon^{1/2} \rho_s^2 \frac{c_s}{L_n} . \tag{3.2.5}$$

Although there may be some modification to this through the details of the fluctuation spectrum, the numerical study described above indicates that their effects will be small.

The regime considered by Parail and Yushmanov (1985) corresponds to shorter-wavelength modes, with

$$k_\perp \sim \omega_{pe}/c . \tag{3.2.6}$$

where c is the speed of light and ω_{pe} is the plasma frequency (the ratio of which corresponds to the skin depth). They consider that stochasticity occurs when

$$\Omega_E \sim \omega_{be} \tag{3.2.7}$$

where ω_{be} is the bounce frequency of trapped electrons. This leads to the test-particle diffusivity

$$\chi_e^{(2)} \sim \epsilon^{1/2} \left(\frac{c}{\omega_{pe}} \right)^2 \omega_{be} . \tag{3.2.8}$$

If both parts of the fluctuation spectrum are present with sufficient amplitude to produce stochastic diffusion a total diffusivity can be written as

$$\chi_e = \epsilon^{1/2} \left\{ \rho_s^2 \frac{c_s}{L_n} \hat{D}_1(\alpha) + \left(\frac{c}{\omega_{pe}} \right)^2 \omega_{be} \hat{D}_2(\alpha) \right\} \tag{3.2.9}$$

where $\hat{D}_m(\alpha)$ represents a slowly varying function of the parameter set $\{\alpha\}$ which defines the fluctuation spectrum. The $\hat{D}_m(\alpha)$ can be treated as adjustable constants of order unity when comparing this formula with experiment. It should be noted that, although the diffusion coefficient was shown to be approximately independent of the fluctuation parameters, it was also shown that there does exist a significant variation with the magnetic shear. Thus, the parameters $\hat{D}_m$ may only be treated as constant at constant shear. Kesner (1989) has analysed the shear variation of the $\hat{D}_m(\alpha)$ obtaining the following fit:

$$\hat{D}_m(\alpha) \to (0.05 + 0.65 e^{-3|s_l|}) \hat{D}_m(\alpha) \tag{3.2.10}$$

where s_l is the local shear on the outside of the torus (i.e. at $\theta = 0$).

The work described so far considers a collisionless plasma and assumes that the trapped-electron diffusion is dominant over that of the passing electrons. If the collisions of electrons with ions are included then trapped electrons are converted to passing at a rate which is proportional to the effective collision frequency of the electrons. This implies that collisions may influence the diffusion coefficient as discussed by Kim *et al* (1990), where the collisional modifications of the Horton *et al* (1987) result are calculated. A similar approach to that of Horton *et al* (1987) is used to derive (numerically) an approximately linear dependence of the diffusion coefficient with the collisionality. Thus Kim *et al* generalize the work of Horton *et al* to

$$\chi_e = \epsilon^{1/2} \left\{ \rho_s^2 \frac{c_s}{L_n} \hat{D}_1(\alpha) + \left(\frac{c}{\omega_{pe}} \right)^2 \omega_{be} \hat{D}_2(\alpha) \right\} + \nu_{ei} \left(\frac{c}{\omega_{pe}} \right)^2 \hat{D}_3(\alpha) . \tag{3.2.11}$$

The new final term is of particular importance at the edge where the collisionality can be high.

Using quasilinear theory Parail and Pogutse (1981) derive an upper bound to the electron thermal diffusivity caused by electromagnetic turbulence which is assumed to exist on a length scale of the collisionless skin depth (c/ω_{pe}). Two expressions for the thermal diffusivity are given for odd and even modes respectively (where 'odd' and 'even' refer to the parity of the electrostatic potential about a resonant surface). Both types of mode are assumed to have the skin depth as their length scale and the difference in the resulting diffusivities is due to the different time steps that are attributed to each. This time step, Δt, is derived by assuming that, in a collisionless plasma, the electrons interact with a wave according to the Landau mechanism, i.e. $\Delta t \sim 1/k_{\|} v_{the}$. For the odd modes, the radial extent of the mode is taken to be $\sim 1/k_\theta$, so that $k_{\|} \sim s/qR$ (where s is the shear, q the safety factor and R the major radius), thus leading to an Ohkawa-type scaling for the electron thermal diffusivity:

$$\chi_e^{(o)} \sim \left(\frac{c}{\omega_{pe}}\right)^2 \frac{v_{the} s}{qR}. \qquad (3.2.12)$$

For even modes the average distance that an electron deviates from the mode rational surface is small and coupling to sidebands needs to be taken into account. This coupling is of order ϵ^2 so that the relevant timescale for this case is $\sim (r/R)^2(v_{the}/qR)$ (where r is the minor radius) leading to

$$\chi_e^{(e)} \sim \left(\frac{r}{R}\right)^2 \left(\frac{c}{\omega_{pe}}\right)^2 \frac{v_{the}}{qR}. \qquad (3.2.13)$$

If both types of mode exist, then the diffusivity due to the odd modes will usually dominate over that of the even modes. However, Parail and Pogutse (1981) claim that the even modes will, as a rule, be excited before the odd modes and thus $\chi_e^{(e)}$ will be the relevant expression.

Finally, in this subsection we consider the work of Zhang and Mahajan (1988). For fluctuations with $\omega < k_{\|} v_{the}$ (where ω is the mode frequency) the relevant timescale is $\sim 1/k_{\|} v_{the}$, thus leading to the result of Parail and Pogutse (1981) described above. Zhang and Mahajan (1988) argue that for this ordering it is not possible for the electromagnetic modes to grow in a collisionless plasma, leading them to propose $\omega > k_{\|} v_{the}$ as the relevant ordering. Then the mode frequency ω provides the timescale for the turbulence. The precise form of the mode which might be responsible for the driving of turbulence is not addressed, but it is assumed that ω scales like $\tilde{\omega}_{*e}$, where

$$\tilde{\omega}_{*e} = \xi \left(\frac{\omega_{pe}}{c}\right) \rho_s c_s \left(\frac{1}{L_{Te}} + \frac{\alpha}{L_n}\right) \qquad (3.2.14)$$

with α and ξ constants, i.e. a linear combination of the diamagnetic drift frequency due to both the temperature and density gradients with the perpendicular wavelength being $\sim c/\omega_{pe}$. Little is said about the constants α and ξ which would be determined once a specific driving mechanism for the electromagnetic turbulence is given; in the absence of such a driving mechanism they must be assumed to be parameters of the model fixed by comparison with experiment. The length scale is taken to be the collisionless skin depth, thus leading to the following form for the diffusivity:

$$\chi_e = \xi \left(\frac{c}{\omega_{pe}}\right) \rho_s c_s \left(\frac{1}{L_{Te}} + \frac{\alpha}{L_n}\right). \qquad (3.2.15)$$

3.3. Electrostatic drift-wave transport

3.3.1. Circulating electron drift wave in a slab or cylinder. The collisionless electron drift wave gives rise to a mode (the 'universal mode') which, in a slab geometry, is always unstable in the absence of (magnetic) shear. In such a case, the radial eigenmode equation possesses a potential well around the maximum density gradient. With the introduction of shear the depth of this well is reduced until, for sufficiently large shear, the potential well becomes a potential hill. A localized mode is no longer possible and it was thought that the universal mode could not exist in such a sheared plasma. However, Pearlstein and Berk (1969) pointed out that a mode whose boundary conditions (far from the mode rational surface) are those of an outgoing wave (i.e. a wave which takes energy from the mode and dissipates it at large distances from the mode rational surface through some mechanism, e.g. ion Landau damping) is an acceptable solution. Thus, in their calculation, there is a competition between the rate of energy radiated by the mode (which increases with shear—the 'shear damping') and the driving mechanism of the instability. For low values of shear (but still typical of tokamak plasmas) the driving mechanism 'wins' and the mode is unstable. Higher shear stabilizes the mode.

An approximation made in the work of Pearlstein and Berk is to expand the plasma dispersion function for $(\omega/k_{\parallel}v_{the})^2 \ll 1$, thus giving rise to an adiabatic electron response. Later work by Ross and Mahajan (1978) and Tsang *et al* (1978) solved the dispersion relation numerically without this approximation and found that shear has a greater stabilizing influence on the 'universal mode' than predicted by the perturbation approach of Pearlstein and Berk. In fact, they found that typical tokamak plasmas had sufficient shear to stabilize the mode. The problem with the Pearlstein–Berk calculation is the breakdown of their approximation near $k_{\parallel} = 0$; the solution in that region has a strong stabilizing influence on the mode.

So far the description of the 'universal mode' has been limited to that of linear theory. Hirshman and Molvig (1979) retain the nonlinear effects arising from the $\boldsymbol{E} \times \boldsymbol{B}$ drift to study the effects of electrostatic turbulence on the mode stability. They argue that the effect of stochastic diffusion of the electron orbits due to this turbulence will generate a finite value for $k_{\parallel}$ (i.e. effectively removing the long wavelength, stabilizing part of the spectrum). Thus it is expected that the turbulence will destabilize the mode relative to the linear predictions. A 'nonlinear' dispersion relation is derived which demonstrates explicitly the destabilizing nature of the turbulence, indicating that quite low levels of turbulence (typically below those expected to exist in tokamaks) are sufficient to drive the mode unstable. In the presence of turbulent fluctuations shear retains its ability to stabilize the mode and there exists a competition between the driving of the turbulence and damping from the shear, with the mode being stable at sufficiently high shear. Solving the dispersion relation at marginal stability determines the diffusion coefficient, D:

$$\chi_e \sim \frac{3D}{2} = \frac{45}{2}\Delta_{PB}^{3/2}\tau^{5/2}\left[\frac{1+\tau}{2}\right]^{-9/2}\frac{c_s\rho_s^2}{L_s} \tag{3.3.1}$$

with

$$\Delta_{PB} = \left(\frac{L_s}{L_n}\right)^3\left(\frac{m_e}{m_i}\right). \tag{3.3.2}$$

Diamond and Rosenbluth (1981) reconsider this problem and find that a low level of turbulence is actually stabilizing. However, at higher levels, more typical of tokamak plasmas, the turbulence is destabilizing in qualitative agreement with Hirshman and Molvig (1979).

3.3.2. Slab-like drift wave in a tokamak. In a slab or cylinder there is a competition between the electron drive (due to Landau resonance in a collisionless regime and collisions at higher collision frequency) and shear damping and that, in fact, the mode is linearly stable. In a tokamak, passing particles continue to provide Landau or collisional drive, as appropriate, but shear damping can be removed by toroidal effects (Taylor 1977). Thus one can use $D \sim \gamma/k_\perp^2$ estimates of the diffusion where the only contribution to γ is the electron drive. Waltz *et al* (1987) present $\gamma/k_\perp^2$ estimates of the transport due to drift waves. Taking $k_\perp\rho_s \sim 1$ they give the following result for the collisionless diffusion coefficient, which is applicable in the region $\nu_e/\omega_{te} < 1$

$$\chi_e \sim D = \frac{q}{\epsilon_n}\left(\frac{m_e}{m_i}\right)^{1/2}\frac{\rho_s^2 c_s}{L_n}. \tag{3.3.3}$$

This clearly differs from the result of Hirshman and Molvig (1979) quoted above. Waltz *et al* also give an expression which is valid in the collisional regime:

$$\chi_e \sim D = \frac{q}{\epsilon_n}\left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{\nu_e}{\omega_{te}}\right)\frac{\rho_s^2 c_s}{L_n} \tag{3.3.4}$$

for $\nu_e/\omega_{te} > 1$. Here, ν_e is the electron collision frequency and $\omega_{te} = v_{the}/(Rq)$ is the electron transit frequency.

3.3.3. Trapped-electron induced modes. In lower collisionality tokamaks trapped-electron effects become important. There then exists a class of drift-type instabilities, the trapped-electron modes, which are more important than the slab–like circulating electron mode which we have considered above. This subsection reviews the extensive literature on the transport that might be expected to result from such modes. There are numerous regimes, mechanisms, mode structures and approaches which have been developed and the theories have been categorized accordingly.

(a) Marginal-stability approach. Manheimer and Antonsen (1979) use a marginal-stability approach to investigate the effect of the dissipative trapped-electron drift instability on temperature profiles. The marginal stability approach assumes that if any part of the plasma becomes unstable then an anomalously high electron transport switches on to return that part of the plasma profile to marginal stability. The model which they use, which does include the effects of shear damping, is greatly simplified (e.g. the density profile is fixed) but the profiles which they obtain are in reasonable agreement with experiment. In this work the stability is determined through numerical solution of the dispersion relation. In an earlier (simplified) calculation (Manheimer *et al* 1976) an analytic stability criterion was derived. Taking the large aspect ratio limit and assuming $\eta_i = 1$ it is:

$$4\left(1+\frac{2T_i}{T_e}\right)\frac{L_{T_e}}{L_s\epsilon^{1/2}} > 1\,.$$

(b) Profile consistency approach. This method for deriving the cross-field transport exploits the fact that the temperature and density profiles in a tokamak are generated by the transport so that a given experimental profile determines the radial dependence of χ_e in terms of the heating source. The overall magnitude of the transport is derived by considering the 'confinement zone' (which is usually taken to be between the $q = 1$ and $q = 2$ surfaces). This avoids detailed discussion of processes in other plasma zones (where the transport is assumed to be very rapid due to, for example, sawteeth at the centre or (possibly) tearing modes at the edge; these processes are partly responsible for the overall shape of the

experimental profile (Waltz *et al* 1986)). For example, Tang (1986) models the experimental temperature profile through the equation

$$T_e(r)/T_e(0) = \exp\left(-\frac{\alpha_T r^2}{a^2}\right) \tag{3.3.5}$$

with α_T chosen to satisfy the empirical relation

$$3\alpha_T/2 \simeq q_a + 0.5 \tag{3.3.6}$$

where q_a represents the edge safety factor. Using the parallel component of Ohm's law with the resistivity $\sim T^{3/2}$ gives

$$j_\parallel(r) = j_0 \exp\left(-\frac{3\alpha_T r^2}{2a^2}\right). \tag{3.3.7}$$

In order to derive the electron thermal diffusivity (for an ohmic tokamak), the ohmic heating is balanced against the transport; thus

$$E_\parallel j_\parallel = -\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left[r\, n_e(r)\, \chi_e(r)\, \frac{\mathrm{d}}{\mathrm{d}r}T_e(r)\right] \tag{3.3.8}$$

so that

$$\chi_e = \chi_{e0} F(r). \tag{3.3.9}$$

All the radial dependence has been absorbed into $F(r)$ which is given by

$$F(r) = \frac{\exp[(\frac{2}{3})(q_a+0.5)(r/a)^2] - \exp[-(\frac{1}{3})(q_a+0.5)(r/a)^2]}{(r/a)^2(n_e(r)/n_e(0))} \tag{3.3.10}$$

and χ_{e0} is to be determined by considering transport due to a particular instability in the confinement zone. Tang postulates that the most important instability that exists between the $q = 1$ and $q = 2$ surfaces is the trapped-electron mode. This leads to an expression for χ_e in this region given by

$$\chi_e \simeq \frac{1}{\sqrt{2}}\left(\frac{m_e}{m_i}\right)^{1/2} \frac{q}{\epsilon_n \nu_{*e}} \frac{[\eta_e + \epsilon_n(1+\eta_e)]}{[1+\bar{c}/\nu_{*e}]} \frac{\rho_s^2 c_s}{L_n}. \tag{3.3.11}$$

The parameter $\bar{c}$ represents the transition from the dissipative to the collisionless mode and should be fixed to be of the order $\bar{c} \sim 0.1$–0.2. Matching volume averages of the entropy production between the $q = 1$ and $q = 2$ surfaces corresponding to the two forms for χ_e leads to

$$\chi_{e0} = 1.6\alpha_n^{0.2} \frac{B_T^{0.3} a^{1.0} Z^{0.2}}{R^{1.9} q_a^{1.6}} \tag{3.3.12}$$

which, together with (3.3.9) and (3.3.10), then gives an expression for the electron thermal diffusivity in an ohmically heated plasma. Here the density profile has been written as

$$n(r)/n(0) = (1-(r/a)^2)^{\alpha_n}. \tag{3.3.13}$$

B_T is the toroidal field in T, temperature is measured in keV, the major, minor radii (R, a) are in m, and the resulting expression for χ_e is in units of $\mathrm{m^2 s^{-1}}$.

Tang also derives the transport that might be expected to exist in a tokamak with auxiliary heating, assuming that the density and temperature profiles of the electrons and ions are the same and, also, the collisionality is restricted to $\nu_{*e} < 0.1$. χ_e is then given by

$$\chi_e = \chi_{eh} F_h(r) \tag{3.3.14}$$

where the radial dependence is in $F_h(r)$:

$$F_h(r) = \frac{\exp[\frac{2}{3}(q_a + 0.5)(r/a)^2]}{(r/a)^2 n(r)/n_0} \frac{\int_0^{r/a} \mathrm{d}x\, x h(x)}{\int_0^1 \mathrm{d}x\, x h(x)} \tag{3.3.15}$$

where $h(r)$ represents the total power deposition profile. The magnitude, χ_{eh} is

$$\chi_{eh} = 0.8 \frac{P_T}{n_0 R T_{i0}(1+\tau)\alpha_T} \tag{3.3.16}$$

where P_T is the total power (in MW) and all other parameters (and units) are as defined above.

(c) Strong turbulence estimates. The $\gamma/k_\perp^2$ models employ the argument that at saturation the growth of the mode (usually taken to be the linear growth rate) will be balanced by turbulent diffusion (i.e. $k_\perp^2 D$). The value of $k_\perp$ is to some extent a free parameter, but experiment seems to favour it to be in the region $k_\perp \rho_s \sim 0.3$. Dominguez and Waltz (1987) use these arguments to derive the electron and ion thermal diffusities that would be caused by ion temperature gradient (ITG) modes, circulating-electron drift modes (both collisional and collisionless) and the trapped-electron modes (collisional and collisionless). Their simplified treatment of the electron modes has a switch from the collisional to the collisionless mode as a collisionality threshold is crossed. (In reality an intermediate collisionality might involve aspects of both modes.) The expressions they give are, for the trapped-electron mode (collisionless or dissipative):

$$\hat{D}_{\mathrm{te}} = \epsilon^{1/2} \frac{\omega_{*\mathrm{e}}}{k_\perp^2} \left\{1, \frac{\omega_{*\mathrm{e}}}{\nu_{\mathrm{eff}}}\right\}_{\min} \tag{3.3.17}$$

for the circulating electron mode (collisionless or dissipative), the result of equations (3.3.3) and (3.3.4):

$$\hat{D}_{\mathrm{ce}} = \frac{\omega_{*\mathrm{e}}}{k_\perp^2} \frac{\omega_{*\mathrm{e}}}{\omega_{\mathrm{te}}} \left\{1, \frac{\nu_{\mathrm{e}}}{\omega_{\mathrm{te}}}\right\}_{\max} \tag{3.3.18}$$

and for the ITG mode a result similar to (2.3.29):

$$\hat{D}_{\mathrm{i}} = \frac{\omega_{*\mathrm{e}}}{k_\perp^2} (2\tau^{-1}\eta_{\mathrm{i}}\epsilon_n)^{1/2}. \tag{3.3.19}$$

Here we have defined the effective collision frequency, ν_{eff} by $\nu_{\mathrm{eff}} = \nu_{\mathrm{ei}}/\epsilon$ and the suggested choice of $k_\perp$ is $k_\perp \rho_s = 0.3$. These transport coefficients are then combined to give total thermal diffusivities of

$$\chi_{\mathrm{e}} = \tfrac{5}{2}(c_{\mathrm{te}}\hat{D}_{\mathrm{te}} + c_{\mathrm{ce}}\hat{D}_{\mathrm{ce}})(1 + \bar{c}_{\mathrm{ei}} f_{\mathrm{i\,th}} \epsilon_n \eta_{\mathrm{i}}) \tag{3.3.20}$$

$$\chi_{\mathrm{i}} = \tfrac{5}{2}[c_{\mathrm{i}}\hat{D}_{\mathrm{i}} f_{\mathrm{i\,th}} + \bar{c}_{\mathrm{ie}}(c_{\mathrm{te}}\hat{D}_{\mathrm{te}} + c_{\mathrm{ce}}\hat{D}_{\mathrm{ce}})] \tag{3.3.21}$$

where the threshold factor for the ion-temperature gradient-driven mode, $f_{\mathrm{i\,th}}$ is given by

$$f_{\mathrm{i\,th}} = \{1 + \exp[-6(\eta_{\mathrm{i}} - \eta_{\mathrm{th}})]\}^{-1} \tag{3.3.22}$$

and η_{th} is a measure of the threshold for the η_{i} mode (see section 2.3).

This expression is interesting because it takes into account the effect of the ion turbulence on the electron heat flow through the coefficient $\bar{c}_{\mathrm{ei}}$ (and the effect of the electron turbulence on ion heat flow through the coefficient $\bar{c}_{\mathrm{ie}}$). The values for the coefficients are to be chosen by fitting to the experimental data. The only constraints on the 'mixing' coefficients given are:

$$1 < \bar{c}_{\mathrm{ei}} < 3 \qquad 0 \leqslant \bar{c}_{\mathrm{ie}} \leqslant 1. \tag{3.3.23}$$

Similar expressions to (3.3.17) are derived by Perkins (1984) using mixing-length estimates for the thermal diffusivities.

An alternative $\gamma/k_\perp^2$ estimate of the electron thermal transport due to the trapped-electron drift mode is given by Romanelli *et al* (1986):

$$\chi_e = \frac{5}{2\sqrt{2}} \frac{q\eta_e}{\epsilon_n \nu_{*e}} \left(\frac{m_e}{m_i}\right)^{1/2} \frac{\rho_s^2 c_s}{L_n} \frac{1}{1+0.1/\nu_{*e}} \tag{3.3.24}$$

where the factor on the right has been included to allow a smooth transition from a collisionless region to collisional. In the dissipative limit this expression yields a factor η_e times the result of (3.3.17).

(d) Weak-turbulence estimate in 'slab' geometry. Gang *et al* (1991) present a weak-turbulence calculation of the transport that is expected from the trapped-electron-driven drift wave. Such a treatment leads to fluctuation levels (and hence transport) which are lower than the predictions of the strong turbulence or mixing-length theories. The geometry is that of the sheared slab and two collisionality regimes are considered: collisionless

$$\omega > \omega_{de} > \nu_{eff} \tag{3.3.25}$$

and dissipative

$$\nu_{eff} > \omega > \omega_{de} \tag{3.3.26}$$

where $\nu_{eff} = \nu_{ei}/\epsilon$. These frequencies are constrained to satisfy

$$\nu_{eff}, \omega_{de}, \omega < \omega_{be} \tag{3.3.27}$$

where ω_{be} is the trapped-electron bounce frequency. A kinetic treatment is employed with the ions described by the nonlinear gyrokinetic equation, the trapped electrons by the nonlinear bounce-averaged drift-kinetic equation and the untrapped electrons assumed to be adiabatic; ion and electron temperature gradients are neglected. The fluctuation spectrum is derived by balancing the linear growth rate, the shear damping and the nonlinear energy transfer. From the fluctuation level and spectrum it is possible to derive the following transport coefficients:

$$\begin{aligned} \chi_e &= 0.8\tau^{-3/2} \left(\frac{L_n}{L_s}\right)^{1/2} \left(A_e^l \frac{\bar{\gamma}}{s}\right)^2 H(\bar{k}_\theta^c) \frac{\rho_s^2 c_s}{L_n} \\ \chi_i &= 0.8\tau^{-3/2} \left(\frac{L_n}{L_s}\right)^{1/2} \left(A_e^l \frac{\bar{\gamma}}{s}\right)^2 F(\bar{k}_\theta^c) \frac{\rho_s^2 c_s}{L_n} \\ D &= 0.4\tau^{-3/2} \left(\frac{L_n}{L_s}\right)^{1/2} \left(A_e^l \frac{\bar{\gamma}}{s}\right)^2 F(\bar{k}_\theta^c) \frac{\rho_s^2 c_s}{L_n}. \end{aligned} \tag{3.3.28}$$

These expressions are valid for both the collisionless and dissipative cases, though the definitions of some of the variables are different in the two collisionality regimes, as outlined below. Thus

$$A_e^l = \sqrt{\epsilon}(L_n/L_s)^{1/2} \ln\left(s\sqrt{\frac{L_s}{L_n}}\bar{k}_\theta\right) \tag{3.3.29}$$

where $\bar{k}_\theta = k_\theta \rho_s$. The variable $\bar{\gamma}$ is a measure of the linear electron drive and is given by

$$\bar{\gamma} = \left\{ \begin{array}{ll} \epsilon_n^{-3/2} e^{-1/\epsilon_n} & \text{collisionless} \\ c_s/(\nu_{eff} L_n) & \text{dissipative.} \end{array} \right\} \tag{3.3.30}$$

The parameter $\bar{k}_\theta^c$ is the cut-off in the wavenumber spectrum due to shear damping. It is obtained by solving the equation which matches the linear electron drive of the mode to the shear damping (i.e. the nonlinear transfer rate is neglected):

$$A_e^l(\bar{k}_\theta^c)\frac{\bar{k}_\theta^c}{s}f(\bar{k}_\theta^c) = \frac{L_n}{L_s} \tag{3.3.31}$$

where

$$f(\bar{k}_\theta) = \begin{cases} \bar{\gamma} & \text{collisionless} \\ \bar{\gamma}\bar{k}_\theta & \text{dissipative.} \end{cases} \tag{3.3.32}$$

The three functions involving this cut-off wavenumber are

$$\begin{aligned} F(x) &= \begin{cases} \frac{5}{7} - x + \frac{2}{7}x^{7/2} & \text{collisionless} \\ \frac{1}{15} - \frac{1}{5}x^3 + \frac{2}{15}x^{9/2} & \text{dissipative} \end{cases} \\ G(x) &= \begin{cases} x^{-1} - \frac{5}{3} + \frac{2}{3}x^{3/2} & \text{collisionless} \\ \ln(x^{-1}) - \frac{2}{3} + \frac{2}{3}x^{3/2} & \text{dissipative} \end{cases} \\ H(x) &= \begin{cases} (1/\epsilon_n)\left[\left(1/\epsilon_n - \frac{3}{2}\right)G(x) - \frac{3}{2}F(x)\right] & \text{collisionless} \\ (1/\pi)[25F(x) + 9xG(x)] & \text{dissipative.} \end{cases} \end{aligned} \tag{3.3.33}$$

A similar theory is given by Rogister (1989) who finds the following form for χ_e:

$$\chi_e \sim \frac{\rho_s^2 c_s}{L_n}\frac{L_{Te}}{L_s}\left(\frac{L_n}{L_s}\right)^2 \frac{T_i}{T_e}\Gamma^* k_\theta \rho_s G(\Gamma^* - 1) \tag{3.3.34}$$

where G is an unspecified function of $(\Gamma^* - 1)$, and $\Gamma^* > 1$ is the condition for the onset of instability. However, this function G increases sufficiently fast with Γ^* that the plasma tends to adopt 'marginal-stability' profiles corresponding to $\Gamma^* = 1$. For $s^2 L_s/L_n > 1$, Γ^* is given by

$$\Gamma^* = \lambda\frac{\epsilon^{1/2}\eta_e}{k_\theta\rho_s}\sqrt{\frac{L_s}{L_n}}\frac{1}{s}\frac{\omega\nu_{\text{eff}}}{(\omega^2 + \nu_{\text{eff}}^2)} \tag{3.3.35}$$

where λ is an adjustable numerical coefficient and $\omega/\nu_{\text{eff}} = 8.86 \cdot 10^{19}\epsilon T_e^2/[A_i^{1/2} n L_n(1 + Z_{\text{eff}})]$. (An expression for Γ^* in the opposite limit is given in Rogister *et al* 1988.) Here T_e is in keV and other parameters are in SI units.

The difference between this result and that presented by Gang *et al* (1991) is partly due to the fact that Gang *et al* neglect the electron temperature gradient; also, they treat the trapped-electrons nonlinearly but the resulting effects are found to be small.

(e) Toroidal mode structure. The nonlinear theory of electron drift waves in a toroidal geometry is considered by Similon and Diamond (1984). When toroidicity is taken into account there exists an extra branch of the drift wave absent in slab geometry (Chen and Cheng 1980). In the slab-like case the radial eigenfunction equation involves a potential hill and the eigenfunctions satisfy the Pearlstein–Berk boundary conditions. When toroidal effects are taken into account the calculation is performed in ballooning space and the corresponding potential hill becomes modified so that there exist local potential wells. These wells are able to confine a new mode—the so-called toroidal drift mode. Of course, the original slab-like mode still exists, though it will be modified slightly due to these local potential wells.

Similon and Diamond suggest that it is this toroidal branch of the drift modes which is important when considering transport and this is the branch which they discuss for a tokamak with circular, concentric flux surfaces. They consider a linear electron response which is taken to be of two parts—an adiabatic piece and a non-adiabatic piece which is modelled by an 'iδ' prescription representing trapped-electron drive. The ions are described by the nonlinear gyrokinetic equation. Calculation of the fluctuation levels and spectrum in an intermediate turbulence regime (corresponding to the turbulent decorrelation rate lying between the ion transit frequency and the linear frequency spectrum width) then leads to the following forms for the test-particle diffusion coefficients. For the untrapped electrons,

$$\chi_e \sim D_u = 0.2\left(\frac{m_e}{m_i}\right)^{2/3}\left(\frac{q}{\epsilon_n}\right)^{4/3}\frac{1}{s^{7/3}\nu_{*e}^{1/3}}\frac{\rho_s^2 c_s}{L_n} \tag{3.3.36}$$

while for the trapped electrons, two collisionality regimes are considered. When $\omega_{be} > \nu_{eff} > \omega_{de}$:

$$\chi_e \sim D_u = 0.2\left(\frac{m_e}{m_i}\right)^{2/3}\left(\frac{q}{\epsilon_n}\right)^{4/3}\frac{1}{s^{7/3}\nu_{*e}^{4/3}}\frac{\rho_s^2 c_s}{L_n} \tag{3.3.37}$$

while for $\nu_{eff} < \omega_{de}$:

$$\chi_e \sim D_t \sim \frac{(2\epsilon)^{1/2}}{2\pi^2}\frac{1}{s^2\epsilon_n}\frac{\rho_s^2 c_s}{L_n}. \tag{3.3.38}$$

A weak-turbulence calculation of the transport due to the toroidal collisionless trapped-electron mode has been performed by Hahm and Tang (1991). They treat both the ions and the trapped-electrons nonlinearly (the circulating electrons are assumed to have a Boltzmann response). The trapped electrons are described by the bounce-averaged drift-kinetic equation and the ions by the gyrokinetic equation. Hahm and Tang argue that the relevant collisionality regime is the collisionless case, and they adopt the ordering:

$$\omega_{*e} > \omega_{De} > \nu_{eff} \qquad \omega_{bi} < \omega < \omega_{be} \tag{3.3.39}$$

where the condition $\omega > \omega_{bi}$ allows the effects of trapped ions to be neglected. The orbit-averaged precession drift frequency, ω_{De}, is defined by $\omega_{De} = \omega_{*e}(\epsilon_n v^2/v_{the}^2)G(s,\,\kappa)$ where the pitch angle κ is related to the turning point of a trapped particle, θ_0, through $\kappa = \sin\theta_0$. For a high aspect ratio tokamak with concentric flux surfaces considered here, $G(s,\,\kappa)$ is given by:

$$G(s,\,\kappa) = 2E(\kappa)/K(\kappa) - 1 + 4s(E(\kappa)/K(\kappa) + \kappa^2 - 1) \tag{3.3.40}$$

where $E(\kappa)$ and $K(\kappa)$ are the complete elliptic integrals of the first and second kind. For the calculation of the fluctuation spectrum $G(s,\,\kappa)$ is replaced by its κ average, $G(s) = 0.64s + 0.57$ (Tang 1990). Using weak-turbulence theory to calculate the fluctuation spectrum, the following, rather complicated, expressions for the transport coefficients are derived. The particle diffusion coefficient, D, is given by

$$D = \left(\frac{2^7\epsilon}{15}\right)\left(\frac{\tau q^2}{s^2}\right)\eta_e\left(1+\frac{5}{4}\eta_i\right)^{-1}\left[\frac{k_M}{k_L} - \ln\left(\frac{k_M}{k_L}\right) - 1\right](k_M\rho_s) \times\left(\frac{R}{L_n}\right)^2\left(\frac{R}{GL_n}\right)^3\left(\frac{R}{GL_n}-\frac{3}{2}\right)^2\exp\left(\frac{-2R}{GL_n}\right)\frac{\rho_s^2 c_s}{L_{Te}} \tag{3.3.41}$$

the electron thermal diffusivity by

$$\chi_e = \left(\frac{2^7\epsilon}{15}\right)\left(\frac{\tau q^2}{s^2}\right)\left(1+\frac{5}{4}\eta_i\right)^{-1}\left[\frac{k_M}{k_L} - \ln\left(\frac{k_M}{k_L}\right) - 1\right](k_M\rho_s)$$
$$\times\left(\frac{R}{L_n}\right)^2\left(\frac{R}{GL_n}\right)^4\left(\frac{R}{GL_n}-\frac{3}{2}\right)^2\exp\left(\frac{-2R}{GL_n}\right)\frac{\rho_s^2 c_s}{L_{Te}} \tag{3.3.42}$$

and, finally, the ion thermal diffusivity by

$$\chi_i = 2.75\frac{(1+1.93\eta_i)}{\eta_i(1+1.25\eta_i)}D\,. \tag{3.3.43}$$

In these expressions

$$k_M^{-2} = \rho_s^2[1+\tau^{-1}(1+\eta_i)]\,. \tag{3.3.44}$$

The wavenumber, k_L is the solution to a rather complicated equation ((33) of Hahm and Tang 1991) but an approximate solution is

$$\left(\frac{k_L}{k_M}\right)^2 \simeq 3(1+\eta_e)G\epsilon_n\,. \tag{3.3.45}$$

(f) Clump theory. Terry and Diamond (1983) and Diamond *et al* (1983) have addressed the role of clumps and their effect on electron drift-wave stability. The authors find that they provide a significant destabilizing influence which can be described in terms of a nonlinear growth rate γ_{NL}, where

$$\gamma_{NL} = \frac{\gamma_L}{[1-C(k,\,\omega_k)]} \tag{3.3.46}$$

with γ_L the linear growth rate and

$$C(k,\,\omega_k) = 2\sqrt{\pi}A(k\rho_s)\frac{\mathcal{S}_k}{\epsilon_n}\left(\frac{\omega_k}{\bar{\omega}_d}\right)^{1/2}\left(1-\frac{\omega_k}{\omega_{*e}}\right)\exp\left(-\frac{\omega_k}{\bar{\omega}_d}\right) \tag{3.3.47}$$

where $A(k\rho_s) = (k^2\rho_s^2)^{-1}[1-J_0(\sqrt{2}k/k_0)]$, $\bar{\omega}_d = \epsilon_n\omega_{*e}$ and k_0^2 is the mean-squared wavenumber. $\mathcal{S}_k$ is a 'shielding response' which Diamond *et al* (1983) give as $\mathcal{S}_k \sim \frac{1}{2}$. The mode frequency, ω_k is a solution of the linear dispersion relation. Taking a mixing-length estimate for the turbulence then leads to the following thermal diffusivity:

$$\chi_e \sim \frac{\gamma_L}{k_\perp^2[1-C(k,\,\omega_k)]}\,. \tag{3.3.48}$$

The results of the linear theory due to Gang *et al* (1991) presented in part (*d*) of this subsection indicate

$$\omega_k = \omega_{*e}/(1+k_\theta^2\rho_s^2) \tag{3.3.49}$$

$$\gamma_L = \frac{|\,\omega_k\,|}{2}\left(\frac{\epsilon}{\pi}\right)^{1/2}\left(1-\frac{\omega_k}{\omega_{*e}}\right)\frac{\Delta}{x_t}\ln\left(\frac{x_t}{\Delta}\right)G_n \tag{3.3.50}$$

where

$$\frac{\Delta}{x_t} = \sqrt{\frac{L_n}{L_s}\frac{1}{s(k_\theta\rho_s)}} \tag{3.3.51}$$

and the value of G_n depends on the collisionality regime:

$$G_n = \begin{cases} 2\sqrt{\pi}(\omega_k/\omega_{de})^{3/2}e^{-\omega_k/\omega_{de}} & \omega_k > \omega_{de} > \nu_{eff} \\ (4/\sqrt{\pi})(|\,\omega_k\,|/\nu_{eff}) & \omega_{de} < \omega_k < \nu_{eff} \end{cases} \tag{3.3.52}$$

(where we note that shear damping does not affect the toroidal mode).

(g) Transport stabilization. Kaw (1982) considers the effects that anomalous thermal transport has on the stability of the collisionless trapped-electron mode. He finds that for sufficiently high values of χ_e the transport has a stabilizing influence on the mode. This suggests a model in which the trapped-electron mode is unstable and drives anomalous transport, which increases until it stabilizes the mode. Kaw illustrates the idea with a relatively simple model of the trapped-electron mode. He considers $\nu_{*e} < 1$ and a fluid description of the ions, with the passing electrons taken to be Boltzmann. The response for the trapped electrons is calculated from a model equation which includes a term describing the effect of the thermal diffusivity on the distribution function. Charge neutrality then leads to a growth rate for the resulting mode of the form:

$$\gamma \sim \epsilon^{1/2} \frac{\omega_{*e}^2}{k_\perp^2 \chi_e} . \tag{3.3.53}$$

It can be seen that χ_e plays a similar role to that of the effective collision frequency in the dissipative trapped-electron mode. χ_e is then assumed to saturate at such a level that the mode will be stable, i.e. this growth of the mode is balanced by the shear damping. This then leads to the following expression for the electron thermal diffusivity:

$$\chi_e = \frac{\alpha \epsilon^{1/2}}{k_\perp^2} \frac{\omega_{*e}}{(1 + k_\perp^2 \rho_s^2)^2} \frac{L_s}{L_{Te}} \tag{3.3.54}$$

where Kaw suggests that $k_\perp \rho_s \sim 1$. The parameter α is an O(1) number describing an averaging over a velocity distribution and can be treated as a fitting parameter.

3.3.4. Inverted gradient profile effects. Inverted gradient profiles (i.e. negative η_e, η_i) have different effects on plasma stability depending on the mode; the circulating-electron drift mode is destabilized relative to the positive $\eta_{i,e}$ case whereas the trapped mode is stabilized (Tang *et al* 1975). One might expect the overall result to be stabilizing when the collisionality is low (and trapped electron effects are important) and destabilizing for high collisionality (when trapped-electron effects are negligible). Horton (1976) investigates this effect by constructing a dispersion relation which describes all three modes (i.e. collisional and collisionless circulating electron modes and the trapped-electron mode). He finds that for $\eta_{i,e} < 0$ there is a gain in stability (over the positive $\eta_{i,e}$ case) when $\nu_{*e} < 0.3$ and a destabilization when $\nu_{*e} > 1$. These linear results are then used to derive a quasilinear estimate of the transport that might be expected from a plasma in which all three instabilities exist. The results are the following, rather complicated, expressions for the particle flux:

$$\frac{\Gamma}{n_0} = -\frac{D_{dw}}{L_n} \left(\Delta S^0 + \eta_e S^1 \right) \tag{3.3.55}$$

and for the electron thermal flux:

$$\frac{q_e}{n_0 T_e} = -\frac{D_{dw}}{L_n} \left(\Delta S^1 + \eta_e S^2 \right) . \tag{3.3.56}$$

In these expressions we have:

$$D_{dw} = \frac{\rho_s^2 c_s}{L_n} \frac{L_s}{L_n} \alpha(k_y \rho_s) \tag{3.3.57}$$

where α is a constant of the order of a few tenths and is to be fitted by comparison with experiment. The parameters S^i represent the following sums:

$$S^i = \sum_{m=1}^{3} G_m^i \tag{3.3.58}$$

where the G^i_m are integrals given in the reference, and $\Delta = \frac{1}{2}$ is suggested.

Considering the dissipative mode ($\nu_{*e} > 1$) Bishop and Connor (1990) simplify Horton's expression to derive

$$\chi_e = \frac{1}{8\sqrt{\pi}}\frac{q}{\epsilon_n \nu_{*e}}\left(\frac{L_s}{L_n}\right)\left(\frac{m_e}{m_i}\right)^{1/2}(15+\eta_e^{-1})\frac{\rho_s^2 c_s}{L_n}. \tag{3.3.59}$$

(Note that this expression differs from that which appears in the main text of Bishop and Connor (1990), where a typographical error has been introduced in transcribing the results derived in their appendix.)

3.3.5. Short-wavelength trapped-electron mode. An analysis of an extremely short-wavelength trapped-electron temperature-gradient driven mode is given by Diamond *et al* (1991). This short-wavelength limit allows the ions to be treated as Boltzmann-like since $k_\theta \rho_s > 1$. An upper bound on k_θ is obtained by requiring that trapped-electron effects are important, i.e. $\omega_r < \omega_{be}$, where ω_r is the real part of the mode frequency. As the mode frequency is of order ω_{*e} the corresponding wavelength range is

$$1 < k_\theta \rho_s < \frac{\epsilon^{1/2}\epsilon_n}{q}\left(\frac{m_i}{m_e}\right)^{1/2}. \tag{3.3.60}$$

Collisions are neglected, which leads to the following constraint:

$$\omega_d > \nu_{eff}. \tag{3.3.61}$$

A linear dispersion relation is derived from the quasineutrality condition, where the passing electrons are also treated as Boltzmann-like and the trapped electrons are described by the bounce-averaged drift kinetic equation. The trapped-electron equation is solved by expansion in powers of ω_d/ω to yield the following dispersion relation for the mode

$$\left[\frac{(1+\tau)}{\sqrt{2\epsilon}} - 1\right]\Omega^2 + \left(\frac{1}{\epsilon_{Te}\eta_e} - \frac{3}{2}\right)\Omega + \frac{3}{2\epsilon_{Te}}\{1/\eta_e + 1\} = 0 \tag{3.3.62}$$

where $\Omega = \omega/\omega_d$ is assumed to be large for the ordering to be consistent. A criterion for instability can be derived from this dispersion relation:

$$\tau > -1 + \sqrt{2\epsilon} + \frac{\sqrt{2\epsilon}}{6}\left(\frac{1}{\epsilon_n} - \frac{3}{2}\right)^2 \frac{\epsilon_n}{(1+\eta_e)}. \tag{3.3.63}$$

For $\epsilon_n \sim \frac{2}{3}$ this is always satisfied and the plasma will be unstable. Equation (3.3.62) can be solved to derive the mode frequency, $\omega_r = F(\epsilon, \tau)\omega_d$ and the growth rate, $\gamma = G(\epsilon, \tau, \epsilon_T)\omega_d$ which can then be used in a 'mixing-length' expression for the electron thermal diffusivity:

$$\chi_e = \frac{2G}{\tau F}\frac{\epsilon_n}{s^2(k_\theta\rho_s)}\frac{\rho_s^2 c_s}{L_n}. \tag{3.3.64}$$

A linear analysis of the short-wavelength limit of the dissipative trapped-electron instability under the same conditions (3.3.60) was performed much earlier by Mikhailovskii (1976). The growth-rate obtained is

$$\frac{\gamma}{\omega} = -\frac{2.6(\bar{\nu}/\omega)^{1/2}(I_1 + \eta_e I_2)}{\{\ln[32(\omega/(2\bar{\nu}))^{1/2}]\}^{3/2}} \tag{3.3.65}$$

where $\bar{\nu} = 3\sqrt{\pi}\nu_{eff}/4$, ω is the mode frequency, given by

$$\omega = 0.58\frac{2\epsilon^{1/2}\omega_{*i}}{1 + T_i/T_e} \tag{3.3.66}$$

and I_1 and I_2 are integrals which can be evaluated numerically with the result that

$$I_1 = 1.61 \qquad I_2 = -1.07\,. \tag{3.3.67}$$

Instability requires

$$\eta_e > 1.52\,. \tag{3.3.68}$$

No expression for the transport is given in this work, but use could be made of the mixing-length expression of (3.3.64), with these values of growth rate and mode frequency. A maximum bound on the diffusivity can be obtained by using $k_\theta \rho_s \sim 1$.

3.4. *Electromagnetic drift-wave transport*

3.4.1. Electromagnetic electron drift-wave turbulence. In a similar vein to their electrostatic drift wave, which we have discussed in section 3.3.1, Molvig *et al* (1979) investigate the effects of turbulence on the electromagnetic electron drift wave in a cylindrical geometry. They find the electromagnetic version is important when the ratio of electron thermal pressure to the magnetic pressure, β_e, exceeds the electron–ion mass ratio, i.e. $\beta_e > (m_e/m_i)$ ($\beta_e = 2\mu_0 p_e/B^2$, where the units are SI). The test-particle diffusion coefficient is then dominated by the magnetic fluctuations. Including this diffusion, D, in the electron-drift–kinetic equation then yields

$$D \sim \chi_e = \frac{0.07}{\tau^{3/2}} \left(\frac{\tau}{1+\tau}\right)^4 \left(\frac{L_s}{L_n}\right)^2 \frac{c_s}{L_n} \left(\frac{c}{\omega_{pe}}\right)^2 \tag{3.4.1}$$

for marginal stability.

3.4.2. Microtearing turbulence. The collisional microtearing mode is linearly unstable in tokamaks but yields very low transport, $\chi_e \sim \nu_{ei}\rho_i^2$, and is unable to account for the observed anomalous transport. In large, high temperature tokamaks, collisionless modes are more relevant, but the collisionless microtearing mode is found to be linearly stable. However, a study by Garbet *et al* (1990a, b) shows that the mode is nonlinearly unstable when the turbulent radial diffusion of the electrons is included. The current perturbation is derived from a modified Vlasov equation for the electrons which incorporates a diffusion term to model these nonlinearities. Inserting the resulting expression for the current into Ampère's law then gives a radial eigenmode equation for the magnetic fluctuations which possesses a potential well characterized by the parameter β_p^*:

$$\beta_p^* = \beta \left(\frac{L_s}{L_n}\right)^2 \left(1 + \frac{\eta_e}{2}\right) \frac{\eta_e}{4}\,. \tag{3.4.2}$$

The assumption that diffusion will occur at a level which renders the mode marginally stable then yields $k_\theta \delta_D$ (where δ_D represents the radial width of the parallel current channel) as a function of β_p^*. This can be approximated by the linear function (Garbet *et al* (1990b))

$$\beta_p^* = 0.25 + 12 \mid k_\theta \delta_D \mid\,. \tag{3.4.3}$$

The electron thermal diffusivity is then given by

$$\chi_e = 6(k_\theta \delta_D) \left(\frac{\delta_D}{\rho_i}\right)^2 \frac{\rho_i^2 v_{the}}{L_s}\,. \tag{3.4.4}$$

The final unknown quantity to be determined is the ratio δ_D/ρ_i which also has a dependence on β_p^*. To evaluate this, one must take electrostatic fluctuations into account; this requires inclusion of ion dynamics for which a Vlasov description is used. Numerical scanning of the parameter space for the marginal-stability condition yields β_p^* as a function of both $k_\theta \rho_i$ and δ_D/ρ_i. Results are quoted for 'typical' tokamak parameters ($q = 2$, $\eta_e = 2$) from which an upper bound to the transport is determined

$$\chi_e \lesssim 0.05 \frac{\rho_i^2 v_{\text{th e}}}{L_s}. \tag{3.4.5}$$

3.4.3. Electron temperature gradient turbulence. The earliest work to suggest that the electron-temperature-gradient driven mode could be the cause of electromagnetic fluctuations and transport in tokamak plasmas is by Rozhanskii (1981). A sheared slab geometry is considered and the hydrodynamic equations are used in two limits—large and small β. In the small β case ($\beta \ll \epsilon^2$) the electron thermal diffusivity is given as

$$\chi_e = \epsilon \left(\frac{c}{\omega_{pe}}\right)^2 \frac{v_{\text{th e}}}{a} \tag{3.4.6}$$

where a is a characteristic length scale in the radial direction. (Appropriate choices of a can be made by comparing with later results to be discussed below. Thus Lee (1987) indicates that one should choose $a \sim L_s$, the shear length scale; whereas the work of Horton *et al* (1988) indicates that $a \sim Rq$ where R is the major radius and q is the safety factor.) In the opposite limit, $\beta \gg \epsilon^2$, the electron thermal diffusivity is

$$\chi_e = \left(\frac{c}{\omega_{pe}}\right) \frac{c_s \rho_s}{a}. \tag{3.4.7}$$

(Comparison with Horton (1988) below suggests that $a = L_{Te}$ is the correct choice.) The next authors to address this electron temperature mode were Guzdar *et al* (1986) who carried out more detailed studies. The theory of the mode in a sheared slab geometry and its implications for the electron transport is described fully in the work of Lee *et al* (1987). Perhaps a more relevant work, as far as tokamaks are concerned, is that of of Horton *et al* (1988) in which the nonlinear properties of this mode are studied in a toroidal geometry. In this subsection we shall discuss these two theories in more detail.

We begin with the sheared-slab treatment of Lee *et al* (1987) which is a more complete description of their original work (Guzdar *et al* 1986). Kinetic theory is used to describe the collisionless electron-temperature-gradient driven mode in terms of a pair of coupled equations in $\tilde{\phi}$ and $\tilde{A}_\parallel$ (which correspond to the electrostatic and electromagnetic fluctuations respectively). The equation is solved numerically to analyse the stability properties of the first three modes (i.e. the three modes with the lowest number of radial nodes). The principal result is that the lowest-order mode is the most unstable and leads to a critical η_e with

$$\eta_{ec} \simeq 1$$

which is consistent with the shorter-wavelength result of Horton *et al* (1988) below. Using quasilinear theory, together with linear properties of the mode derived from the numerical analysis, they derive the following expression for the electron thermal diffusivity:

$$\chi_e \sim 0.13 \left(\frac{c}{\omega_{pe}}\right)^2 \frac{v_{\text{th e}} s}{qR} f(\eta_e) \tag{3.4.8}$$

where $f(\eta_e) = \eta_e(1 + \eta_e)$. This result is restricted to

$$\eta_e(1 + \eta_e) \ll \frac{69}{\tau}. \tag{3.4.9}$$

Connor (1988) uses scale-invariance arguments to demonstrate the scaling of (3.4.8) for $\eta_e \to \infty$ when $f(\eta_e) \to$constant.

The theory of Horton *et al* (1988) mainly employs hydrodynamic equations in a tokamak with a circular cross-section. Hydrodynamic theory predicts that there exists a critical value for η_e

$$\eta_{e,c} \simeq \tfrac{2}{3} \tag{3.4.10}$$

above which there is an instability. Inclusion of FLR effects raises this value to

$$\eta_{e,c} \simeq 1\,. \tag{3.4.11}$$

These can be reconciled by kinetic theory which indicates that the two results correspond to different perpendicular wavelength regimes:

$$\eta_{e,c} \sim \begin{cases} \tfrac{2}{3} & \text{for } k_\perp \rho_{ei} \lesssim 0.3 \\ 1 & \text{for } k_\perp \rho_{ei} \sim 0.5\,. \end{cases} \tag{3.4.12}$$

thus indicating that FLR effects become important when $k_\perp \rho_{ei} \gtrsim 0.3$ where

$$\rho_{ei} = \left(\frac{T_i}{m_e}\right)^{1/2} \frac{1}{\Omega_{ce}} \tag{3.4.13}$$

with Ω_{ce} the electron cyclotron frequency.

The linear theory indicates that the most important η_e driven modes are those with short wavelength and are electrostatic in nature. Under the restrictions of the validity of the hydrodynamic treatment, i.e.

$$s/q < 1/(2\epsilon_{T_e})^{1/2} \tag{3.4.14}$$

the toroidal regime

$$s < 2q \tag{3.4.15}$$

and the condition that the density profile be sufficiently peaked

$$\epsilon_n < 1\,. \tag{3.4.16}$$

Horton *et al* derive the following mixing-length estimate for this short-wavelength mode:

$$\chi_e = \left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{2q}{s}\right) \eta_e^{1/2} \frac{\rho_s^2 c_s}{\tau L_n}\,. \tag{3.4.17}$$

In the limit of flat density gradient (i.e. $\eta_e \gg 1$) they give

$$\chi_e = \left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{2q}{s}\right) (2\epsilon_{T_e})^{1/2} \frac{\rho_s^2 c_s}{\tau L_{T_e}}\,. \tag{3.4.18}$$

Both of these exhibit a rather low level of transport.

They then proceed to show that the longer-wavelength part of the spectrum develops an electromagnetic component. In Horton *et al* (1987) it is shown that these longer-wavelength electromagnetic perturbations can give rise to substantial transport when the motion becomes stochastic in nature. Linear theory indicates that in the case of the η_e driven mode there is marginal stability in this long-wavelength limit. However, in the nonlinear theory a long-wavelength mode can be driven unstable through an interaction with two shorter-wavelength modes. Stochastic diffusion then leads to two scalings for the transport coefficient, corresponding to the two situations where stochastic diffusion occurs

(Horton *et al* 1987). For the regime where the circulation frequency of the vortices, Ω_E, is comparable to the mode frequency they derive

$$\chi_{1,e} \sim \left(\frac{c}{\omega_{pe}}\right)\frac{c_s\rho_s}{L_{T_e}} \tag{3.4.19}$$

where the typical scale length for the turbulence has been taken to be the collisionless skin depth, c/ω_{pe}. In the other regime, where the circulation frequency is comparable to the bounce frequency of the trapped electrons, they derive:

$$\chi_{2,e} \sim \epsilon^{1/2}\left(\frac{c}{\omega_{pe}}\right)^2\omega_{be}\,. \tag{3.4.20}$$

They are therefore lead to the following result for the electron thermal diffusivity due to stochastic electromagnetic drift waves driven by electron temperature gradients:

$$\chi_e \sim \left(\frac{c}{\omega_{pe}}\right)\frac{c_s\rho_s}{L_{T_e}} + \epsilon^{1/2}\left(\frac{c}{\omega_{pe}}\right)^2\omega_{be}\,. \tag{3.4.21}$$

Neither of these obey the $\eta_e(1+\eta_e)$ scaling predicted in (3.4.8) by Lee *et al* (1987). This is because of the hydrodynamic treatment employed by Horton *et al* which imposes $\eta_e \gg 1$; as shown by Connor (1988) $f(\eta_e) \rightarrow$constant in this limit.

3.5. Conclusions

Electron transport due to electron drift-wave turbulence is a mature subject and a great variety of electron thermal diffusivities have been generated in the literature. A test of these against JET data has been carried out by Tibone *et al* (1994). Many of the electrostatic drift wave models fail to produce radial profiles for χ_e increasing towards the plasma edge, although more success is achieved by invoking the marginal stability idea. The models based on the collisionless skin-depth turbulence are found to produce somewhat better radial profiles. All the models we have discussed are of the gyro-Bohm family corresponding to fluctuations on the scale of ρ_s or c/ω_{pe}. However, it should be noted that the recent numerical simulations of electrostatic drift waves in a sheared slab by Carreras *et al* (1992) have indicated longer wavelength fluctuations around low-order rational surfaces which could lead to Bohm-like behaviour. Connor *et al* (1993) have shown that toroidal coupling effects could also lead to extended drift-wave structures. The associated transport may therefore be large, possibly causing the density, temperature, etc to take up marginally stable profiles; in any case one might again expect a more Bohm-like scaling.

4. Electron transport induced by magnetic islands

4.1. Overview

The magnetic component of electromagnetic fluctuations considered in section 3 can cause stochastic magnetic fields and result in an anomalous transport according to the Rechester and Rosenbluth (1978) formula. In this section we concentrate on the magnetic fluctuations associated with finite-sized magnetic islands resulting from nonlinear tearing-mode island instabilities. This topic has seen rapid growth recently and therefore warrants a more detailed discussion.

A tearing instability associated with a given rational surface leads to a chain of magnetic islands centred on that surface. In a tokamak, many such rational surfaces exist and one can envisage a situation when the island chains overlap and interfere with each other. When this occurs the magnetic field becomes stochastic in the region of overlap and anomalous transport is expected (Rechester and Rosenbluth 1978).

In the linear regime the tearing-mode amplitude (and corresponding island width) grows exponentially with time, before rapidly entering a nonlinear regime in which the growth becomes linear in time (Rutherford 1973):

$$\frac{\partial w}{\partial t} \sim \frac{\Delta'}{\mu_0 \sigma}. \tag{4.1.1}$$

Here w is the island width, Δ' is the standard tearing-mode parameter and σ is the plasma conductivity. In the limit that the poloidal mode number $m \gg 1$, Δ' is negative ($\Delta' \approx -m/r$) and (4.1.1) implies micro-islands cannot form. One has to identify a destabilizing mechanism to drive the islands against this damping and a variety of these exist as we shall discuss in detail in section 4.2. Balancing such a drive (which typically depends on w) against the Δ' damping then gives rise to an expression for the steady-state island width. Many of the theories which we shall discuss rely on the very presence of the island for the drive to exist. This, together with the stochasticity condition gives rise to two bounds on the island widths. First the islands must have a sufficiently large width that they overlap with the island chain on the adjacent rational surface (the so-called stochasticity condition). Second the island width must not be so large as to completely destroy all surrounding islands (and so lose self-consistency of the theory).

Let us now turn to the level of transport which is expected when islands of the required width exist in the plasma. In the stochastic region we can employ the results of Rechester and Rosenbluth (1978) for the test-particle diffusion coefficient

$$D = v_{\mathrm{th\,e}} \sum_{m,n} L_c \left(\frac{\delta B_r}{B}\right)^2 \tag{4.1.2}$$

where L_c is the correlation length. The fluctuation amplitude can be expressed in terms of the island width w, shear length L_s and minor radius r, i.e.

$$\frac{\delta B_r}{B} = \frac{m w^2}{16 r L_s}. \tag{4.1.3}$$

White and Romanelli (1989) evaluate L_c in terms of the island width to be

$$L_c = \frac{2\sqrt{2} R q}{n q' w}. \tag{4.1.4}$$

Thus, given the island width (which in general will depend on m) one can derive an expression for D (which we assume to be representative of the electron heat diffusivity). Strictly one should sum over all n (and $m = nq$) for which the islands exist. However, assuming that the mode spectrum is dominated by some typical wavenumber m leads to the following simple expression for χ_e:

$$\chi_e \sim \frac{v_{\mathrm{th\,e}} w^3 m}{L_s r}. \tag{4.1.5}$$

Knowledge of m and w, which depends on the particular driving mechanism, then leads to a scaling for the diffusivity. This treatment of the transport may be too simplistic in that it does not address the bounds on w which we discussed earlier (and is therefore more relevant when islands evolve independently).

White and Romanelli (1989) describe a theory of the transport which *does* address these bounds. Although they consider a particular driving mechanism for the islands, their treatment is generic to all models which predict the existence of stable islands in a stochastic sea. The degree of stochasticity is quantitatively described by defining a 'stochasticity' parameter α_s such that

$$\alpha_s = \sum_n nq'w \tag{4.1.6}$$

with

$$\alpha_{sc} < \alpha_s < \alpha_{st} \tag{4.1.7}$$

corresponding to the region of validity. The lower bound α_{sc} (~ 1) must be exceeded for islands to overlap, whereas the upper bound, α_{st} indicates stochasticity at such a high level that all magnetic islands will be destroyed. The sum over toroidal mode number, n, in the definition of α_s is assumed to be bounded by some value, $n = N$. Taking the island width to scale as $w = \bar{w}/n$ then gives a simple expression for α_s:

$$\alpha_s = \bar{w}Nq' . \tag{4.1.8}$$

This equation determines the value of $\bar{w}$ for which transport is important. Using equations (4.1.2)–(4.1.4) together with (4.1.8) then leads to the expression

$$\chi_e \sim \frac{v_{\mathrm{th\,e}}R}{q^3s^2}\left(\frac{r}{R}\right)^2\left(\frac{\alpha_s}{N}\right)^3 . \tag{4.1.9}$$

To obtain a result which can be compared with experiment, values for α_s and N need to be chosen. We have described above how transport is only important when islands partially overlap (but are not completely destroyed); this suggests the choice $\alpha_s \sim 1$ (corresponding to marginal stochasticity). The number of modes N could be treated as a constant in order to obtain a simple scaling for χ_e with the plasma parameters. If N is indeed a constant, then (4.1.9) describes transport for any mechanism which leads to islands in regions of a stochastic magnetic field. A variety of such mechanisms exist as we shall describe in the next subsection.

Finally in this subsection we mention the semi-empirical, but successful, Rebut–Lallia–Watkins transport model (Lallia *et al* (1988) and Rebut *et al* 1989). This model combines an assumption that transport due to magnetic island formation switches on when a critical temperature gradient is exceeded, with empirical results from power-balance studies to suggest a scaling for the electron thermal diffusivity (which is constructed in a dimensionally correct form). Chains of magnetic islands are assumed to exist, localized around rational surfaces although the detailed mechanism for their creation is not addressed in these first works. A chaotic region exists between the island chains when there is overlapping of the islands and the existence of such a region leads to enhanced transport. This is characterized by a stochasticity parameter, above which islands overlap and regions of stochastic magnetic field exist between them. It is argued that this happens when the electron temperature gradient exceeds a critical value. This critical value and the anomalous diffusivity are obtained from inspection of Ohmic, L-mode and H-mode JET data.

Four dimensionless parameters are chosen to describe the dominant physical processes occuring in the tokamak—plasma pressure is represented by $\beta_p = 2\mu_0 p/B_\theta^2$, resistivity by $S = \eta J/(B_\theta v_{th})$, diamagnetic drift by $\Omega = (\nabla kT/eB_\theta v_{th})^2$ and power flow by $\Phi = P/(3\pi^2 rRnkTv_{th})$. JET data then suggests the following scalings for the critical temperature gradient and heat flux

$$\Omega_{ec} = S_e/\beta_{pe} \qquad \Phi_e = \Omega_i^{1/2}S_e^{1/2}(1-(\Omega_{ec}/\Omega_e)^{1/2}) \tag{4.1.10}$$

where the subscripts i, e represent electron and ion quantities respectively. One can also utilise certain dimensionless geometrical factors such as aspect ratio, safety factor, etc. When these are taken into account a critical temperature gradient of

$$(\nabla kT_e)_c = 0.06\left[\frac{\eta J B^3}{n_e (kT)^{1/2}}\right]^{1/2}\left(\frac{1}{q}\right)\left(\frac{e^2}{\mu_0 m_e^{1/2}}\right)^{1/2} \tag{4.1.11}$$

is postulated (in SI units and with k being the Boltzmann constant). Note that η is the classical Spitzer resistivity and other parameters are standard. This implies

$$(\nabla T_e) \propto \frac{\epsilon^{7/4}\nu_*^{1/2}}{\beta_e q^2}\frac{T_e}{a}. \tag{4.1.12}$$

The anomalous electron heat flux, F_e, is taken to scale as

$$F_e = n_e\chi_e\nabla(kT_e)\left[1-\left|\frac{\nabla T_e^c}{\nabla T_e}\right|\right]$$

$$\chi_e = 0.15\left[\frac{1}{L_{Te}}+\frac{2}{L_{ne}}\right]\frac{\tau^{1/2}}{\epsilon}\left(\frac{q^2}{q' B R^{1/2}}\right)c^2(\mu_0 m_i)^{1/2}. \tag{4.1.13}$$

when the temperature gradient exceeds the critical value given in (4.1.11) and the radial derivative of the safety factor satisfies $q' > 0$ ($F_e = 0$ otherwise). Under the same conditions, the following expression for the anomalous ion heat transport is obtained:

$$F_i = n_i\chi_i\nabla(kT_i)\left[1-\left|\frac{\nabla T_e^c}{\nabla T_e}\right|\right]$$

$$\chi_i = \frac{\chi_e Z_i \tau^{1/2}}{\sqrt{1+Z_{\rm eff}}} \tag{4.1.14}$$

where the $Z_{\rm eff}$ scaling is introduced in the later work by Taroni *et al* (1991). A scaling for the particle flux is also obtained in this later work:

$$D_e \propto \chi_e\left[1-\left|\frac{\nabla T_e^c}{\nabla T_e}\right|\right]. \tag{4.1.15}$$

It is interesting to note that one can express χ_e in the gyro-Bohm form

$$\chi_e \sim \left(\frac{r}{R}\right)^{3/4}\frac{L_s}{L_n}\left(1+\frac{\eta_e}{2}\right)\left(\frac{\nu_{*e}}{\beta_e}\frac{\tau}{q}\right)^{1/2}\frac{\rho_s^2 c_s}{R} \tag{4.1.16}$$

or equivalently,

$$\chi_e \sim \left(\frac{r}{R}\right)^{3/4}\frac{L_s}{L_n}\left(1+\frac{\eta_e}{2}\right)\left(\frac{\nu_{*e}}{q}\frac{m_i}{m_e}\tau\right)^{1/2}\left(\frac{c}{\omega_{pe}}\right)\frac{\rho_s c_s}{R}. \tag{4.1.17}$$

4.2. Island drive mechanisms

The essential feature of theories of island growth is that they should give rise to a perturbed parallel current which is able to drive the island growth against the Δ' damping illustrated in (4.1.1). As the expression for the transport given in (4.1.9) is independent of the island drive, we expect this result to be applicable to all the theories discussed in this subsection. Alternatively, one can use the expression (4.1.5), substituting for the corresponding island widths.

4.2.1. Stochastic transport of current density. White and Romanelli (1989) examine whether current, carried from one island chain to another by diffusion across the stochastic region, could self-consistently sustain the islands (i.e. counteract the natural stability of the high m tearing mode). Particles can diffuse radially across the stochastic region and (assuming the collisionality to be small) they will carry information from one island chain and deposit it at another after a correlation time $\tau_c = L_c/v_{th}$ where L_c is the correlation length (see (4.1.4)). The distance travelled in this time, L, is related to the test-particle diffusion coefficient, D, through $L = (D\tau_c)^{1/2}$. The net current fluctuation, $\delta J_\parallel$, arriving at an island chain from two adjacent chains (a distance L away) is given by $\delta J_\parallel = L^2 J''_\parallel$ where $J_\parallel$ represents the equilibrium current and primes denote radial derivatives. This perturbed current can only result when the stochastic region exists; it then leads to island growth if $J''_\parallel > 0$ and this drive exceeds the decay due to Δ'. One then finds that stable islands can exist if the parameter, p, where

$$p = \frac{6Ns^2 q B_\phi}{\pi^2 \mu_0 R r^2 J''_\parallel} \tag{4.2.1}$$

lies in the range

$$0 < p < p_c \tag{4.2.2}$$

where the value of p_c is not determined but can be taken to be approximately unity. With increasing drive, p decreases to zero and α_s approaches a stable solution α_c (where $\alpha_c \sim 1$) to maintain the estimate (4.1.9). In fact, the solution for α_s approaches α_c very rapidly as p drops below p_c so that the transport is expected to be relatively independent of the drive once the threshold has been passed. One might expect a similar threshold to exist whatever the driving mechanism.

4.2.2. Pfirsch–Schlüter and bootstrap current islands. A modification to (4.1.1) due to Pfirsch–Schlüter-type currents can be obtained by taking into account the combined effects of magnetic curvature and pressure gradients. This work has been performed within the framework of MHD by Kotschenreuther *et al* (1985) where it is found that (4.1.1) is modified by a term, α_p, proportional to the pressure gradient, i.e.

$$\frac{\partial w}{\partial t} \sim \frac{1}{\mu_0 \sigma}\left(\Delta' + \frac{\alpha_p}{w}\right). \tag{4.2.3}$$

From (4.2.3) we see that if $\alpha_p > 0$ then Pfirsch–Schlüter currents are able to drive high m perturbations and stable small-scale islands of width $w = -\alpha_p/\Delta'$ can form. This corresponds to an equilibrium with unfavourable average magnetic curvature whereas a tokamak (which has favourable curvature) has $\alpha_p < 0$. Thus Pfirsch–Schlüter-type currents cannot drive small-scale islands in a tokamak for $q > 1$.

In a tokamak particles can become trapped in the weaker magnetic field on the outboard side. These can couple to the pressure gradient and produce a current which flows parallel to the field lines—the so-called 'bootstrap current'. The difference in bootstrap currents inside and outside a magnetic island might sustain the island. This is addressed by Carrera *et al* (1986) who consider a toroidal equilibrium at large aspect ratio. A collisionality parameter, $d = \nu_e/\omega_{te}$, is defined, where ν_e is the electron collision frequency and ω_{te} is the transit frequency. The motion around the major axis is assumed to be collisionless, leading to the conditions

$$d < \epsilon^{3/2} < 1\,. \tag{4.2.4}$$

Within the island the motion is collisional and, since the 'island collisionality' $\sim (r/w)\nu_{*e}$, the island width is ordered as $w \sim d^2 a$ where a is the plasma minor radius. The current is calculated from the Ohm's law for the island, which Carrera *et al* derive from a solution of the nonlinear drift kinetic equation for the electrons. Two small expansion parameters are identified—d (as defined earlier) and $\delta = \rho_e/w$ where ρ_e is the electron Larmor radius and w is the island width. Expanding to zeroth order in δ and third order in d leads to the lowest-order Ohm's law in the island region:

$$J_\parallel = 2.3(1 - 2.1\epsilon^{1/2})\sigma_s \bar{E}_\parallel \tag{4.2.5}$$

where ϵ is the inverse aspect ratio of the flux surface about which the island chain is centred, $\bar{E}_\parallel$ is the poloidal average of the electric field in the island and σ_s is the classical Spitzer conductivity, $\sigma_s = 1.97ne^2\tau_e/m_e$. The total conductivity is reduced relative to the classical value due to the effect of trapped particles and this modifies the Δ' damping term.

In order to obtain the bootstrap current contribution it is necessary to expand the drift kinetic equation to first order in δ. This results in the following expression for the island bootstrap current:

$$J_b = -\frac{0.73}{I_0 I_2}\epsilon^{1/2}\rho_{pe} e v_{the}\frac{dn}{dr} \tag{4.2.6}$$

where $I_{0,2}$ are numerical integrals (defined in the reference), and ρ_{pe} is the poloidal electron Larmor radius (temperature gradient effects have been neglected which is justified in that they contribute relatively little to the bootstrap current—at least in the large aspect ratio equilibrium case). Combining this with (4.2.5) leads to the total current, from which the following island width can be deduced:

$$w = 2.3\frac{(1 - 2.1\epsilon^{1/2})}{q\Delta'}\epsilon^{3/2}\beta_p\frac{L_s}{L_n} \tag{4.2.7}$$

where L_n is the density scale length (retaining the sign). Note that for hollow density profiles ($L_n > 0$) or negative shear ($L_s < 0$) bootstrap current islands cannot form.

Kuvshinov *et al* (1989) generalize the work of Kotschenreuther *et al* (1985) and Carrera *et al* (1986) (discussed above) to include finite pressure effects by considering a tearing parity resistive ballooning mode as a possible candidate for magnetic islands in a tokamak. Stability of these pressure-gradient driven modes is characterized by a parameter Δ'_B to be found from the asymptotic form for the solution in the (linear) ideal region (the subscript B indicates that the quantity is calculated in ballooning space). In the region around the rational surface they solve the equations describing continuity, longitudinal equation of motion (neglecting inertia), Ampère's law and the longitudinal Ohm's law (including the bootstrap current). This gives an expression for magnetic perturbations in the 'layer' (related to Δ'_B through matching to the ideal region) from which the following equation for the island evolution can be obtained:

$$\frac{\partial w}{\partial t} \sim \frac{1}{\mu_0\sigma}\left\{\Delta'_B + \frac{32G_2}{w}\left(\frac{\mu_e\alpha}{4\epsilon\nu_e s} - \frac{1}{2}(U_0 + H)\right)\right\} \tag{4.2.8}$$

where G_2 is a numerical coefficient ($G_2 \simeq 0.39$). Other parameters are $\alpha = -\epsilon\beta_p r/L_p$ and the longitudinal electron viscosity, μ_e; U_0 is a measure of the average curvature (defined such that $U_0 < 0$ if the average curvature is good) and $H = \epsilon\beta(\epsilon + \alpha)/s$. The terms proportional to $1/w$ represent the effects of bootstrap and Pfirsch–Schlüter currents. Strauss (1981) has shown that $\Delta'_B < 0$ if $2H^{1/2} \ll 1$. In a tokamak both H and U_0 are small compared to the bootstrap current term (proportional to μ_e in (4.2.8)), especially in the low-collisionality regime $\nu_{*e} \ll 1$, and one may therefore drop these terms when evaluating the steady-state island width. In this low-collisionality regime $\mu_e \simeq \nu_e\sqrt{\epsilon}$ so that (using the Strauss result for Δ'_B) the steady-state island width for the tearing parity resistive ballooning mode is given by

$$w = -12.5\frac{r}{m}\left(\frac{1}{\epsilon s}\right)^{1/2}\tan\left(\frac{\epsilon r\beta_p}{8L_p s^{1/2}}\right). \tag{4.2.9}$$

In the limit $\beta_p \to 0$ the scaling of (4.2.7) is recovered (though there is a small discrepancy in the numerical coefficient).

4.2.3. Drift and FLR effects. Smolyakov (1989, 1993) investigates the possibility that drift and FLR effects may cause a perturbed parallel current. The small-scale islands which result are thus termed 'drift magnetic islands'. Considering a sheared-slab equilibrium geometry (relevant if the island width is much less than the plasma radius) Smolyakov calculates the evolution of magnetic islands in two limits—$\rho_i \ll w$ and $\rho_i \gg w$. In each case a parallel current is evaluated from the continuity equation:

$$\nabla_\perp \cdot (n\boldsymbol{v}_E + n\boldsymbol{v}_{de} - \boldsymbol{\Gamma}_i) = \frac{1}{e}\nabla_\parallel \cdot \boldsymbol{J}_\parallel \tag{4.2.10}$$

where $\boldsymbol{v}_E$ is the $\boldsymbol{E}\times\boldsymbol{B}$ drift, $\boldsymbol{v}_{dj}$ is the diamagnetic drift, $\boldsymbol{\Gamma}_i$ is the transverse ion flux and the subscript j labels the species. The contributions to the ion flux depend on the ion Larmor radius regime under consideration.

In the regime $\rho_i \gg w$ the large ion orbits average over the electrostatic potential and the ions take up a Boltzmann distribution so that $\boldsymbol{\Gamma}_i$ vanishes. The electron density and the potential are calculated from the collisional Braginskii equations (continuity, Ohm's law and temperature equation) and quasineutrality. These are then used to evaluate the left-hand side of (4.2.10) from which the parallel current perturbation $\delta J_\parallel$ can be calculated. The two equations which result from matching the real and imaginary parts of the island layer to the ideal layer (through Ampère's equation) then lead to expressions for the steady-state island width, w and its rotation frequency, ω:

$$w = -\frac{0.25}{\Delta'}\frac{L_s^2}{L_n^2}\frac{\eta_e}{\eta_{cr}}\left[2-\frac{\eta_e}{\eta_{cr}}\right]\beta \tag{4.2.11}$$

$$\omega = \omega_{*e}(1-\eta_e/\eta_{cr}). \tag{4.2.12}$$

Here, electron and ion temperatures have been assumed equal (i.e. $\tau = 1$) η_e is the ratio of electron density scale length to temperature scale length, $\eta_{cr} = 0.49$ and $\beta = 2\mu_0 p/B^2$. For high m modes (so that $\Delta' \simeq -m/r$) we can see that there will be two situations which lead to instability (island growth):

$$\eta_e > 2\eta_{cr} \qquad \text{or} \qquad \eta_e < 0 \tag{4.2.13}$$

(where $\omega_{*e} > 0$, $\omega_{*i} < 0$). In the first case the islands are predicted to rotate in the ion diamagnetic drift direction and in the second in the electron diamagnetic drift direction.

In the opposite limit, $\rho_i \ll w$, fluid equations for the ions are appropriate. The ion continuity equation is used to determine the ion density perturbations with contributions from the ion polarization velocity and gyroviscosity included. Following a treatment analagous to the short-wavelength limit Smolyakov derives the following expression for the stable island width:

$$w^3 = 1.1\frac{\rho_i^2}{\Delta'}\frac{L_s^2}{L_n^2}\left[1-\frac{\eta_e}{\eta_{cr}}\right]\left[2+\eta_e-\frac{\eta_e}{\eta_{cr}}\right]\beta. \tag{4.2.14}$$

The rotation frequency is the same as that for short-wavelength modes (i.e. equation (4.2.12)) and the condition to be satisfied for islands to exist is

$$\eta_{\rm cr} < \eta_{\rm e} < \frac{2\eta_{\rm cr}}{1-\eta_{\rm cr}} . \tag{4.2.15}$$

Hugon and Rebut (1991) attempt to justify the critical-temperature-gradient model of Rebut *et al* (1989) discussed in section 4.1 by proposing a similar mechanism for self-sustainment of magnetic islands. Two contributions to island evolution are considered—one due to the different responses of electrons and ions to the islands (because of their differing Larmor radii) and one due to an artificial 'gravity' (which plays the role of magnetic curvature). The model is analysed in a slab magnetic geometry with cylindrical effects incorporated through the gravity. This models unfavourable magnetic curvature and leads to island growth; in a tokamak (for $q > 1$) the favourable average magnetic curvature tends to damp the islands. We shall, therefore, not consider the curvature effects in this model (see section 4.2.2 for the effects of curvature in a tokamak) but instead describe the differing Larmor radius effects which have been discussed in more detail in Rebut and Hugon (1991). These arise when the ion Larmor radius is comparable with the island width. The electrons have a small Larmor radius and respond to the local electrostatic potential associated with the island as they move along magnetic flux surfaces. (These are closed within the islands but open beyond the island separatrix, leading to different electron responses in the two regions.) The ion gyro-orbit, however, samples both inside and outside the island so the ions have a non-local response to the potential. Thus the electrons and ions move under different effective potentials and therefore they have different $\boldsymbol{E} \times \boldsymbol{B}$ drifts. This gives rise to a current perpendicular to the magnetic field which is not divergence free. In order to satisfy $\nabla \cdot \boldsymbol{J} = 0$ a parallel current must flow and it is this current flow which leads to island growth. Stable islands are formed when this growth balances the stabilizing Δ'.

The calculation involves the radial electric field which is determined by an ambipolarity condition for transport in the stochastic region. This field is expressed in terms of the equilibrium density and temperature gradients by taking a Boltzmann electron density distribution and assuming the electrons travel along the perturbed magnetic field lines with their thermal velocity. This gives rise to the result:

$$E_0 = \frac{T_{\rm e}}{e}\left(\frac{n_{\rm e}'}{n_{\rm e}} + \frac{1}{2}\frac{T_{\rm e}'}{T_{\rm e}}\right) . \tag{4.2.16}$$

More formally, one can make a quasilinear estimate of the electron flux using a kinetic description. Requiring that this flux be zero (to first order in the electron–ion mass ratio) leads to the following expression for the electric field (Samain 1984):

$$E_0 = \frac{T_{\rm e}}{e}\left\{\frac{n_{\rm e}'}{n_{\rm e}} + \left[\left(\frac{\omega}{k_\| v_{\rm the}}\right)^2 - \frac{1}{2}\right]\frac{T_{\rm e}'}{T_{\rm e}}\right\}$$

which reduces to (4.2.16) for fast electrons, such that $\omega \sim k_\| v_{\rm the}$.

Expressions for the island width can be obtained in the two limits $\rho_{\rm i} \gg w$ and $\rho_{\rm i} \ll w$. For $\rho_{\rm i} \gg w$

$$w \sim \frac{2}{\Delta'}\frac{L_{\rm s}^2}{L_n^2}\eta_{\rm e}\left[1+\frac{\eta_{\rm e}}{4}\right]\beta \tag{4.2.17}$$

and for $\rho_{\rm i} \ll w$

$$w^3 \sim \frac{\rho_{\rm i}^2}{\Delta'}\frac{L_{\rm s}^2}{L_n^2}\left[1+\frac{\eta_{\rm e}}{2}\right]\left[2+\frac{\eta_{\rm e}}{2}\right]\beta \tag{4.2.18}$$

for $\tau = 1$. It is interesting to note that these results are very similar to those obtained by Smolyakov (equations (4.2.11) and (4.2.14)) despite the different analytic approaches employed and the different physical assumptions that have been made (in particular, Smolyakov's model has collisional electrons while in the Rebut–Hugon model they are treated as collisionless). Comparison of the results shows that they exhibit the same scalings except that the two differ in their coefficients for the η_e terms. This therefore implies different threshold criteria for the islands to exist. In particular, the Rebut–Hugon model predicts islands to exist for all positive η_e, while the Smolyakov model requires η_e to lie within a certain range (which can include negative η_e) given by (4.2.13) and (4.2.15). Samain (1984) obtains almost the same result as (4.2.17) but with a different numerical coefficient (0.3 instead of 2).

The resulting transport in this Rebut–Hugon model is assumed to be important only when adjacent islands overlap, i.e. when a region of stochastic magnetic field exists between the islands. This overlap condition corresponds to a critical temperature gradient which is most easily reached for $\rho_i \gg w$. For $\beta \ll 1$ this condition can be written:

$$L_{Te}^2 < \frac{q\beta R L_s}{2}$$

Overlap for islands with $w \lesssim \rho_i$ may be more significant for transport and this requires a larger critical temperature gradient. This could be obtained from a numerical solution of the full Rebut–Hugon expression for the island width if desired.

Smolyakov and Hirose (1993) have considered the collisionless case when electron inertia replaces resistivity in the Ohm's law. Thin islands such that $w \sim c/\omega_{pe} \ll \rho_i$ are assumed; thus the ion density perturbation is described by a Boltzmann relation. Fluid theory is used to describe the electron response to the magnetic and electrostatic perturbations, from which the perturbed current sustaining the island against the Δ' damping can be evaluated. This gives rise to an expression for the saturated island width in terms of the rotation frequency, ω, of the island. The dominant contribution to the matching condition which determines ω comes from the resonant region $k_\parallel v_{the} \sim \omega$. Because of the assumed thin islands, this lies far outside the island in a region where linear theory is valid. Thus linear theory can be used to determine the rotation frequency, $\omega = (1 + \eta_e/2)\omega_{*e}$ and the island width follows:

$$w^2 = \left(\frac{c}{\omega_{pe}}\right)^2 \frac{1 + \eta_e/2}{\eta_e} \tag{4.2.19}$$

Assuming $k_\theta \sim w^{-1}$, equation (4.1.5) determines the electron thermal diffusivity:

$$\chi_e \approx \left(\frac{c}{\omega_{pe}}\right)^2 \frac{v_{the}}{L_s} \frac{1 + \eta_e/2}{\eta_e}. \tag{4.2.20}$$

4.2.4. Thermal effects on island drive. Thermal instabilities can also lead to magnetic island formation as a result of a modification of the island resistivity through temperature perturbations (Rebut and Hugon 1985; Dubois and Mohamed-Benkadda 1991). This then gives rise to a perturbed current which may drive the island. The electron temperature in the island is given by the power-balance equation for the electrons; thus the net power increase, P_v, resulting from Ohmic and additional heating, is balanced against losses due to heat conduction, radiation and electron–ion heat transfer. Assuming thin islands ($mw/r \ll 1$) Rebut and Hugon derive an equation for the electron temperature perturbation δT_e in terms of P_v taking the temperature dependence of P_v to be weak, i.e.

$$\frac{\mathrm{d}P_v}{\mathrm{d}T_e}\delta T_e < \frac{P_v}{2}. \tag{4.2.21}$$

Assuming a resistivity variation $\sim T_e^{3/2}$ then leads to an expression for the current perturbation in the layer. Matching to the external solution leads to the following expression for the saturated island width (for $m \gg 1$)

$$w = \frac{mr}{M}\frac{\sqrt{R^2q^2 + r_0^2}}{Rq} \tag{4.2.22}$$

where

$$M = -\frac{3}{2}\mu_0\frac{J_0 r}{B}\frac{P_v}{\chi_{\perp e}^{\mathrm{neo}}T}\frac{Rq^2}{q'} \tag{4.2.23}$$

with J_0 the equilibrium current. The diffusivity $\chi_{\perp e}^{\mathrm{neo}}$ is the electron diffusivity within the island (assumed to be neoclassical). The condition for islands to form is that $M > 0$ which, for $q' > 0$ (the usual case), corresponds to $P_v < 0$, i.e. the sinks exceed the sources. This situation is most likely to occur outside the $q = 2$ surface where radiation losses are largest.

In the work of Dubois and Mohamed-Benkadda (1991) the theory of Rebut and Hugon (1985) is extended to allow for the fact that stochasticity in the vicinity of the island separatrix reduces the effective size of the island. These authors also use a more realistic model of the impurity radiation term in the power-balance equation. Specifically, they allow for a temperature dependence of this term (modelled through a quadratic dependence of the radiative emissivity on the temperature) which then permits stable islands on a smaller scale than those predicted by Rebut and Hugon. Energy balance yields a somewhat more complicated equation for the island width requiring numerical solution. Two solutions for the width result—a low unstable one and a higher stable one.

4.2.5. Nonlinear ion pumping in a torus. We now discuss this model due to Kadomtsev (1991) which proposes another mechanism for anomalous transport involving magnetic islands filling the tokamak. The theory is distinct from those discussed in this section so far because Kadomtsev does give expressions for χ_e and these differ from that derived by White and Romanelli (1989) (i.e. equation (4.1.9)). The basic idea of the theory is that when an ion in its Larmor orbit, radius ρ_i, traverses a much smaller scale magnetic island it acquires a transverse impulse (i.e. directed within the average magnetic surface) from electrostatic potentials associated with the island. This impulse produces a velocity ΔV where

$$m_i\Delta V \sim k_\theta e\tilde{\phi}\frac{w}{V_\perp} \tag{4.2.24}$$

and $w/V_\perp$ is the transit time across the island. This causes a radial shift $\Delta V/\omega_{ci}$ in the guiding centre. During the time $\Delta t \sim \omega_*^{-1}$ that the island remains in phase there are multiple crossings which produce a net shift

$$\delta_0 \sim \frac{\omega_{ci}}{\omega_*}\frac{e\tilde{\phi}}{m_i V_\perp}\frac{k_\theta w}{\omega_{ci}} \tag{4.2.25}$$

due to a single island. However the ion interacts with $N = \rho_i/w$ neighbouring islands during its orbit which will produce uncorrelated shifts. These are therefore considered as a sequence of random shifts so that their net effect is given by:

$$\delta_1 = \delta_0 N^{1/2}. \tag{4.2.26}$$

The island pattern due to N similar waves with the mean frequency ω_* and a similar spread $\Delta\omega \sim \omega_*$ will be quasi-periodic on a longer period Ω^{-1} where $\Omega = \omega_*/N$. During this time the shifts can again be summed in a random fashion to yield a total radial shift

$$\delta \sim N^{1/2}\delta_1 \sim w \tag{4.2.27}$$

Thus the ion can step in a stochastic manner from one island chain to another.

Since the period Ω is so long, this is a slow process and the magnetic moment μ is conserved; in an homogeneous field its energy is therefore also conserved. In an inhomogeneous toroidal field, however, a shift of the ion position to larger major radius can liberate energy to pump the islands. Over many periods Ω^{-1} the shifts will time average to zero. However, a mechanism for a systematic shift exists if the islands are slanted with respect to the mean magnetic surface by an angle γ, which can arise if the island structures are touching each other. Because the islands are slanted, a component of its electric field lies along the radial direction producing a poloidal shift $\gamma\delta$. This prolongs the interaction time with the island and $\Delta t \to (1 + \gamma k_\theta \delta_0)\omega_*^{-1}$, resulting in

$$\delta_0 \to \delta_0(1 + \gamma k_\theta \delta_0)\,. \tag{4.2.28}$$

A systematic radial shift $\langle\delta\rangle \sim \gamma k_\theta \delta_0^2$ follows from averaging the quadratic term. The corresponding flux as the ion intersects the N neighbouring islands in the time $\Delta t \sim \omega_*^{-1}$ is

$$\Gamma \sim n\omega_* N\langle\delta\rangle \sim \frac{\gamma}{L_n} k_\theta^2 w^3 v_{\text{thi}} n \tag{4.2.29}$$

directed along the major radius. To obtain a net flux accross a toroidal surface and island pumping it is necessary to take $\gamma \sim \gamma_0 \epsilon \cos\theta$ so that averaging over a toroidal surface element produces

$$D \sim \gamma_0^2 \epsilon k_\theta^2 w^3 v_{\text{thi}}\,. \tag{4.2.30}$$

Simultaneously the island is pumped at a nonlinear rate

$$\frac{\partial E}{\partial t} = F_{\text{i}} \frac{r}{R^2} k_\theta^2 w^2 v_{\text{thi}} n T \tag{4.2.31}$$

where $F_{\text{i}} \sim \gamma_0 w/L_n$.

We turn now to the electron dynamics which are described by a simplified drift–kinetic equation. Kadomtsev argues that for low collisionality plasmas, dissipation effects on a single island structure are not important and the island is free to grow due to the ion pumping above. The electron motion takes the form of drift islands displaced from the magnetic islands. As the magnetic island grows the drift islands grow at the same rate and eventually overlap so that the electron motion becomes stochastic, allowing a transfer of longitudinal momentum between magnetic islands. Kadomtsev interprets this as an anomalous resistivity, with an anomalous 'collision frequency', $\nu_a \sim k_\| v_{\text{the}} \sim (v_{\text{the}}/qR)k_\theta w$. Because of the increase in dissipation with w, the island width will saturate at some value. This can be quantified by balancing the ion 'pumping' growth against the electron dissipation damping,

$$\frac{\partial E}{\partial t} = -\frac{v_{\text{the}}}{Rq}\frac{w^3 k_\theta B_\theta^2}{8\pi r^2} F_{\text{e}} \tag{4.2.32}$$

where F_{e} is a measure of the island overlap.

Predictions for anomalous thermal diffusivity, $\chi_{\text{e}} \sim \nu_a w^2$, are made in two regimes—weak island contact and strong island overlap. Thus

$$\chi_{\text{e}} = \Phi_{\text{e}} k_\theta w^3 \omega_{\text{te}} \tag{4.2.33}$$

where the degree of overlap is represented by a factor Φ_e. In the weak island contact limit the island pumping by the ions is assumed to be small so that the electrons are able to saturate the width at a relatively low value. An estimate of Φ_e is made as follows. In toroidal geometry stochastization occurs in a layer near the island separatrix of width $\sim \epsilon w$. An exchange of electrons within this layer then gives rise to a radial heat flux, which is most efficiently transferred by the slow passing electrons (i.e. $\sim \epsilon^{1/2}$ of all electrons in the layer) leading to an overall scaling $\epsilon^{3/2}$. The island width is assumed to scale like the collisionless skin depth (c/ω_{pe}) and k_θ is assumed to satisfy $k_\theta w \sim 1$, so that

$$\chi_e \sim \epsilon^{3/2}\omega_{te}\left(\frac{c}{\omega_{pe}}\right)^2 . \tag{4.2.34}$$

Stronger transport results when the islands have much more contact. Then the island width is obtained by balancing the island pumping by ions against the damping by electrons to give, again for $k_\theta w \sim 1$,

$$\frac{w^2}{r^2} \sim \frac{qr}{R}\beta_\theta\sqrt{\frac{m_e}{m_i}} . \tag{4.2.35}$$

This yields the following form for the thermal diffusivity

$$\chi_e = C_e\frac{v_{thi}}{R^2}r^3\beta_\theta \tag{4.2.36}$$

where it is supposed that C_e is only weakly dependent upon the plasma parameters.

4.3. Conclusions

The Rechester and Rosenbluth formula provides an expression for the diffusion arising from a spectrum of magnetic fluctuations. These magnetic fluctuations can arise from fine scale magnetic islands and a variety of nonlinear driving mechanisms have been discussed, a number of which are proportional to β. A generic form (4.1.9) for the Rechester–Rosenbluth diffusivity arising from such islands was obtained. This has neither a Bohm or gyro-Bohm scaling if the number of toroidal modes, N, is treated as a constant; however, N may depend on $\rho_* = \rho_i/a$ and a somewhat novel scaling could emerge. The ion-pumping mechanism of Kadomtsev also leads to a χ_e scaling essentially independent of ρ_*. On the other hand the semi-empirical formula of Rebut–Lallia–Watkins is gyro-Bohm but it would be desirable to generate a more sound theoretical basis for it.

Testing of these models against JET data (other than the well advertised tests of the Rebut–Lallia–Watkins model) was carried out by Tibone *et al* (1994). The ion-pumping model provided the best radial profiles and parameter scalings.

The Rechester and Rosenbluth formula suggests that the rapid electrons will escape more readily than the ions, leading to a positive radial electric field (in the rest frame in which the magnetic fluctuations appear stationary). This is indeed the case for externally produced magnetic perturbations but for self-consistent fluctuations satisfying Maxwell's equations quasilinear theory indicates that transport is automatically ambipolar (Waltz 1982); the radial electric field is determined by viscosities instead.

Finally we note that some of the mechanisms encountered in driving laminar magnetic islands in this subsection (e.g. bootstrap currents) also play a role in fully developed 'fluid' turbulence theories which we describe in section 5.

5. Resistive fluid turbulence

5.1. Overview

In this section we consider the instabilities of a plasma described as a 'resistive' fluid, noting that 'resistive' is to cover both neoclassical effects and the consequences of electron inertia in Ohm's law. We shall see that transport can result from two sources: turbulent mixing and a 'stochastic' radial diffusion. The latter is as a consequence of the magnetic fluctuations which are predicted to arise because of the instability. These can interact to form stochastic magnetic fields, and a subsequent radial transport by parallel motion. In the following subsections we shall consider two types of resistive fluid instability: resistive pressure-gradient driven modes in section 5.2 and resistivity-gradient driven modes in section 5.3. Some conclusions follow in section 5.4.

5.2. Resistive-pressure-gradient driven transport

In this subsection we consider the transport which would result because of instability to a resistive-pressure-gradient driven mode. In the limit of cylindrical geometry, where there is unfavourable curvature, the mode is unstable and has an interchange nature (and is thus termed the resistive interchange mode); it is relevant for a description of the transport in RFP's or stellarators where there is bad average curvature. In a tokamak the average curvature is 'good' when $q > 1$ and therefore this mode is usually stable. However, tokamaks are truly toroidal, requiring a full toroidal treatment of the pressure-gradient driven mode. The mode then has a ballooning nature and the bad curvature region can dominate. This (resistive ballooning) mode may be unstable in a tokamak and is the instability on which we concentrate in this subsection.

The equations describing a plasma as a resistive fluid are invariant under certain sets of scaling transformations of the various plasma parameters. This invariance can be used to determine the dependence of the diffusivity on these parameters. In fact, if sufficient assumptions are made about the equations which govern the turbulence evolution then a complete scaling of the diffusivity can be derived. This has been done for the resistive pressure-gradient driven mode (Connor and Taylor 1984) where two contributions to the transport are considered—a convective cross-field diffusion and a loss due to parallel transport along the stochastic magnetic field. The turbulence is considered in the following limits:

$$n \gg 1 \qquad n^2/S \ll 1 \qquad \beta q^2/\epsilon < 1 \tag{5.2.1}$$

where n is the toroidal mode number, $S = \tau_R/\tau_A$ (with the resistive diffusion time, τ_R and the poloidal Alfvén time, τ_A, defined by $\tau_R = \mu_0 r^2/\eta$ and $\tau_A = (\mu_0\rho_m)^{1/2}Rq/B$ respectively, where η is the plasma resistivity and ρ_m is the mass density). Assuming that the diffusion coefficient scales as the square of a radial step size to a time step, the invariance transformations lead to the following result for the convective diffusion coefficient:

$$D_c = g_0 \frac{\eta}{\mu_0}\left(\frac{\alpha}{s}\right) \tag{5.2.2}$$

where g_0 is a constant factor and

$$\alpha = -\frac{2\mu_0 Rq^2}{B^2}\frac{\mathrm{d}p}{\mathrm{d}r}. \tag{5.2.3}$$

The radial transport due to parallel diffusion along the stochastic magnetic field lines is given by (4.1.2). Two collisionality regimes may be considered—highly collisional (such that $\lambda_{\rm mfp} < L_{\rm c}$) and collisionless (such that $\lambda_{\rm mfp} > L_{\rm c}$). For the collisional case the dominant mechanism for decorrelation of the electron paths is collisions, so that $D_1 = (v_{\rm the}^2/\nu_{\rm e})(\delta B/B)^2$. Using the invariance transformations of the equations describing the turbulence:

$$\left(\frac{\delta B}{B}\right)^2 = g_1 \frac{\epsilon^2}{q^2 S}\left(\frac{\alpha}{s}\right)^{5/2} \tag{5.2.4}$$

where g_1 is a numerical coefficient. This gives the following result for the diffusion due to the stochasticity of the magnetic field lines in the collisional limit:

$$D_1 = g_1 \frac{v_{\rm the}^2}{\nu_{\rm e}} \frac{\epsilon^2}{q^2 S}\left(\frac{\alpha}{s}\right)^{5/2} . \tag{5.2.5}$$

In the collisionless case, the electron path is correlated over the magnetic-field-line correlation length $L_{\rm c}$. This is calculated by assuming that the diffusion of the magnetic field lines is described by a random walk whose characteristic step length, Δr, is given by

$$\Delta r = (\delta B_r/B)L_{\rm c} \tag{5.2.6}$$

and deriving the forms of Δr and $(\delta B_r/B)$ using the scaling arguments. Substitution into (4.1.2) then leads to the following result for the stochasticity diffusion coefficient in the collisionless limit:

$$D_2 = g_2 \frac{R\epsilon^2 v_{\rm the}}{qS}\left(\frac{\alpha}{s}\right)^{3/2} . \tag{5.2.7}$$

Renormalized quasilinear calculations (Carreras *et al* (1983b)) are of necessity in agreement with these diffusion coefficients.

Subsequently Carreras *et al*(1987) considered the nonlinear calculation of the diffusion resulting from the resistive interchange mode. The equations are similar to those considered by Connor and Taylor (1984) apart from two extra terms appearing in the pressure evolution and vorticity evolution equations which are included in order to model the viscosity μ and cross-field thermal diffusivity $\chi_\perp$. A linear analysis of these equations indicates that these two terms are crucial in identifying a saturation mechanism for the nonlinear analysis. It is found that for non-zero values of both μ and $\chi_\perp$ there exists a critical poloidal mode number $m_{\rm c}$ above which the plasma is stable to the resistive interchange mode. Saturation occurs when the rate of transfer of energy from the unstable low m modes to the stable high m modes equals the rate of increase of energy driving the instability. Renormalization techniques are used to represent the nonlinearities in terms of a 'nonlinear' diffusion coefficient and viscosity so that the equations resemble the linear expressions with enhanced transport coefficients. Solution of the equations leads to an eigenvalue expression which, for zero growth rate, gives the following relation between the nonlinear diffusion coefficient and viscosity:

$$D = D_{\rm ml}\Lambda \tag{5.2.8}$$

where $D_{\rm ml}$ represents a mixing-length expression, i.e. the cylindrical analogue of $D_{\rm c}$ (see (5.2.2)). The enhancement Λ is a logarithmic factor (dependent on the viscosity and D) which enters wholly as a result of the introduction of the renormalized diffusion and viscous terms and can lead to a significant increase in the predicted diffusion over mixing-length estimates (without significantly altering the scaling).

Of course, resistive interchange modes are of little relevance to tokamaks but they are closely related to the resistive ballooning mode. Comparison with the results of Connor and

Taylor (1984) suggests the following form for the enhancement factor Λ in the case of the resistive ballooning mode:

$$\Lambda = \frac{2}{3\pi} \ln \left[\frac{64 \alpha r^4}{m^4 \tau_A^2 \mu_{nl} D} \right] . \tag{5.2.9}$$

The nonlinear viscosity, μ_{nl} is related to the diffusion, D through

$$\mu_{nl} = \left(\frac{m^2}{\langle m^2 \rangle} \right)^{1/2} \frac{D}{\Lambda} \tag{5.2.10}$$

and

$$D = D_c \Lambda \tag{5.2.11}$$

where D_c is the result of (5.2.2). An iterative solution of (5.2.11) yields an expression for D in terms of the plasma parameters:

$$D = \frac{2}{3\pi} g_0 \frac{\eta}{\mu_0} \frac{\alpha}{s} \ln \left(\frac{64 r^4}{m^4 q^2 \tau_A^2} \left[\left(\frac{\langle m^2 \rangle}{m^2} \right)^{1/2} \frac{\mu_0^2 s^2}{g_0^2 \eta^2 \alpha} \right] \right) . \tag{5.2.12}$$

This expression involves the poloidal mode number m ($\langle m^2 \rangle^{1/2}$ is its RMS value) and a value for this needs to be chosen. Numerical simulation indicates that the most important m value is such that $m = \langle m^2 \rangle^{1/2}$ and that $\langle m^2 \rangle^{1/2}$ varies with the parameter $\beta/(2\epsilon^2)$. Carreras *et al* (1987) quote a table of m values as a function of $\beta/(2\epsilon^2)$

$\beta/(2\epsilon^2)$	$\langle m^2 \rangle^{1/2}$
0.0025	12
0.0050	6
0.0075	3
0.0100	3
0.0125	3 .

(5.2.13)

The heat transport due to stochastic magnetic fields set up by instability to resistive pressure-gradient-driven turbulence is analysed by Carreras and Diamond (1989) using the Rechester and Rosenbluth formula. The 'collisionless' limit is used and an expression for the correlation length, L_c, in terms of the magnetic fluctuations is invoked. The magnetic fluctuations are related to those in the electrostatic potential through Ohm's law. Assuming the spectrum of poloidal wavenumbers again satisfies $m = \langle m^2 \rangle^{1/2}$ then yields the electron thermal diffusivity due to resistive ballooning modes:

$$\chi_e = \frac{1}{2^{13/6}} \frac{1}{\langle n^2 \rangle^{1/3}} \frac{1}{S^{2/3}} \frac{q^2}{s} \left(\beta \frac{R_0}{L_p} \right)^{4/3} \frac{v_{the} r^2}{R_0} . \tag{5.2.14}$$

Here $\langle n^2 \rangle^{1/2}$ represents an rms average of the toroidal mode number for the instability, and is related to the poloidal wavenumber through $m = nq$.

Diamagnetic effects modify χ_e (Carreras *et al* (1983a)) through the substitution

$$\chi_e \rightarrow \chi_e \left(\frac{n_c^2}{\langle n \rangle^2 + n_c^2} \right) \tag{5.2.15}$$

where

$$n_c = \frac{\beta^2 q^3}{s \epsilon_0^2} \left(\frac{L_n}{L_p} \right)^2 \left(\frac{a}{\rho_s} \right)^3 \left(\frac{L_n}{a} \right) \left(\frac{a}{v_{the} \tau_A} \right)^3 \frac{a}{r} \tag{5.2.16}$$

with $\epsilon_0 = a/R$, a being the plasma minor radius.

The thermal diffusivity due to the resistive ballooning mode in a magnetic geometry which has a separatrix is calculated by Hahm and Diamond (1987). Considering a simple equilibrium where the separatrix has two X-points, they investigate how such a geometry affects the diffusivity. An identification of the saturation mechanism for the turbulence is required and this is assumed to occur when the pressure fluctuations are such that they mix the pressure gradient over the radial mode width. The following expression for the thermal diffusivity due to stochasticity of the magnetic field is obtained:

$$\chi_e = f_\chi(\rho)\chi^0 . \tag{5.2.17}$$

Here χ^0 is the value of the diffusivity in a circular flux surface geometry:

$$\chi^0 = \frac{3}{2} v_{\mathrm{th}e} r \frac{1}{S}\left(\frac{\alpha_\psi}{s_c}\right)^{3/2} \tag{5.2.18}$$

and $f_\chi(\rho)$ represents the effects of the flux surface shaping:

$$f_\chi(\rho) = \left(\frac{s}{s_c}\right)^{-3/2}\left(\frac{q}{q_c}\right)^{3/4}\left[3^{2/3}\left(\frac{E-(1-\rho^2)K}{(1+\rho^2)E-(1-\rho^2)K}\right)^{2/3}\right]^{9/4} \tag{5.2.19}$$

where $K(\rho)$ and $E(\rho)$ are the complete elliptic integrals of the first and second kind:

$$K(\rho) = \int_0^1 \frac{\mathrm{d}t}{\sqrt{(1-t^2)(1-\rho^2t^2)}} \qquad E(\rho) = \int_0^1 \mathrm{d}t\sqrt{\frac{1-\rho^2t^2}{1-t^2}} \tag{5.2.20}$$

with ρ a radial parameter, $\rho = r/r_s$ where r is the minor radius, measured along the symmetry plane ($\theta = 0$) and r_s is the value of r at the separatrix surface. The safety factor q has the usual definition and q_c is the cylindrical limit. The shear parameters are defined as $s = \mathrm{d}\ln q/\mathrm{d}\ln\rho$ and $s_c = \mathrm{d}\ln q_c/\mathrm{d}\ln\rho$. Other parameters are the pressure gradient, α_ψ

$$\alpha_\psi = -\frac{2\mu_0 r_0^2}{B_{p0}}\frac{\mathrm{d}p}{\mathrm{d}\psi} \tag{5.2.21}$$

and $S = \tau_R/\tau_A$ as defined previously. Here, ψ is the poloidal flux function, so that this definition of α_ψ reduces to that of (5.2.3) in the circular flux surface limit.

So far we have described the pressure-gradient-driven turbulence through the resistive MHD equations, which are strictly only valid in the short-mean-free-path (Pfirsch–Schlüter) regime. At lower collisionalities (in the banana or plateau regimes) trapped-particle effects can become important through their viscous interaction with the circulating particles. For example, in the presence of a pressure gradient, this viscous interaction gives rise to an additional current, the so-called bootstrap current. Connor and Chen (1985) use gyrokinetic theory to derive the linear stability properties of the pressure-gradient-driven mode in a low collisionality plasma. Even in the absence of curvature instability is possible due to the bootstrap current and neoclassical viscous damping. Because of its role in the driving mechanism, this mode is sometimes referred to as the 'bootstrap current mode'. An alternative linear analysis of the bootstrap current mode was given by Callen and Shaing (1985) using a set of modified fluid equations that take into account neoclassical effects. These give the same result for the linear growth rate as that derived using gyrokinetic theory by Connor and Chen. The nonlinear theory and resulting transport has been investigated by Kwon *et al* (1990) using this set of neoclassical MHD equations. The saturation mechanism for the turbulence is similar to that described previously—only low m modes are unstable and as the mode grows, the nonlinear terms transfer energy to the stable higher m modes where it can be dissipated. To describe such a saturation mechanism a full two-point theory would have to be employed. Kwon *et al* instead use a one-point method whereby the nonlinear terms are renormalized to give turbulent diffusivities. Saturation is then assumed to occur when the diffusivities reach such a level that the growth of the fluctuations vanishes. This leads to an eigenvalue equation, which can be solved to yield the following expression for the pressure diffusivity:

$$D_p = \frac{1}{8\pi}\frac{\epsilon}{q}\beta_p\frac{\eta}{\mu_0}\delta_e\frac{L_s}{L_p}\Lambda_n^2 \tag{5.2.22}$$

with

$$\Lambda_n^2 = 3\delta_e^{-1} - 1 + 2[\delta_e^{-1}(2\delta_e^{-1} - 1)]^{1/2} \tag{5.2.23}$$

and

$$\delta_e = \frac{\mu_e/(\alpha_e\nu_e)}{[1 + \mu_e/(\alpha_e\nu_e)]} \tag{5.2.24}$$

in terms of the following definitions:

$$\mu_e = \frac{2.3\sqrt{\epsilon}\nu_e}{[1 + 1.02\nu_{*e}^{1/2} + 1.07\nu_{*e}]} \tag{5.2.25}$$

$\alpha_e \simeq 0.51$, $L_p = -(\mathrm{d}\ln p/\mathrm{d}r)^{-1}$ and β_p is the poloidal beta. This neoclassical instability also gives rise to stochasticity of the magnetic field, which can therefore result in enhanced electron heat transport

$$\chi_e = 4.6 \times 10^{-2} v_{\mathrm{th\,e}} L_s \left[\frac{\epsilon}{q}\beta_p\right]^{4/3} \delta_e^{5/3} \left[\frac{r^2}{mL_p^2}\right]^{2/3} S^{-2/3}\Lambda_n^{7/3} \tag{5.2.26}$$

where m needs to be specified.

The expressions above are controlled by collisional (or neoclassical) resistivity. Itoh *et al* (1993) invoke a model involving anomalous electron viscosity in the Ohm's law. Introducing an anomalous fluid viscosity and thermal diffusivity in the vorticity and thermal equations respectively they obtain an unstable ballooning mode. Assuming that the anomalous transport coefficients are all due to turbulence associated with this instability they can be related through their quasilinear expressions. Their values when the corresponding turbulence is saturated can be obtained by demanding that the most unstable mode is marginally stable. The result for the fluid thermal diffusivity is

$$\chi = \frac{v_A}{qR}\left(\frac{c}{\omega_{pe}}\right)^2 \frac{\alpha^{3/2}}{h(s)} \tag{5.2.27}$$

where $h(0) = 1.7$, $h(s > 0.7) = 2.5s$. This can be considered as an analogue of the work of Carreras *et al* (1987) for a collisionless fluid.

Recently Connor (1993) has pointed out that the anomalous transport coefficients can arise from renormalizations of the electron inertia in the collisionless Ohm's law. Using invariance techniques Connor obtains a similar result to (5.2.27) and also an expression for the associated stochastic magnetic field electron thermal diffusivity:

$$\chi_e = \frac{v_{\mathrm{th\,e}}}{Rq}\left(\frac{c}{\omega_{pe}}\right)^2 \frac{\alpha^2}{s}. \tag{5.2.28}$$

These forms are valid for fluid electrons with $\alpha > m_i\beta_e/m_e$. In hotter plasmas where the reverse inequality holds, a kinetic Ohm's law is required. Using the Ohm's law suggested by Kadomtsev and Pogutse (1985) which incorporates electron Landau damping, Connor obtains

$$\chi = \frac{v_{\mathrm{the}}}{Rq}\left(\frac{c}{\omega_{\mathrm{pe}}}\right)^2 \alpha \tag{5.2.29}$$

and

$$\chi_{\mathrm{e}} = \frac{v_{\mathrm{the}}}{Rq}\left(\frac{c}{\omega_{\mathrm{pe}}}\right) \rho_{\mathrm{s}} \frac{\alpha^{3/2}}{s^{1/2}} \tag{5.2.30}$$

for fluid turbulent convection and electron transport due to the associated stochastic magnetic fields, respectively.

5.3. Resistivity-gradient driven transport.

Resistivity-gradient driven turbulence can result from two sources: a gradient in the electron temperature or a gradient in Z_{eff} (i.e. an impurity density gradient). The instability caused by an electron temperature gradient resulting in a radial variation of the resistivity is often called the rippling mode. This is a mode which is of a resistive MHD nature and is driven by a radial gradient in the current (which exists as a consequence of the resistivity gradient). A high electron collisionality is necessary in order to overcome the stabilizing influence of the parallel electron thermal conduction, and this would imply that this mode could be relevant to transport at the tokamak edge. The linear stability of the rippling mode in a sheared slab is investigated by Hassam and Drake (1983) using Braginskii fluid equations with a modified Ohm's law (Hassam 1980). The modification is due to the time-dependent thermal force which exists in the presence of a temperature gradient. This term is neglected in Braginskii's equations which are valid only in the limit that the parallel diffusion rate for the electrons is larger than the mode frequency. Hassam's modified fluid equations contain this extra drive and so are useful for investigations of the linear stability of the rippling mode. Hassam and Drake (1983) identify three regions in parameter space. In region I, at very low temperature, the mode is described by an Ohm's law in which parallel pressure perturbations are small compared to the resistivity perturbation, and then the mode is found to be purely growing in nature. As the temperature rises the plasma enters region II where the pressure perturbations become more important, with density fluctuations being described by the electron continuity equation. The mode frequency then gains a real part which is of the order of the diamagnetic drift frequency but the mode is still unstable. As the temperature is raised still higher, the collisionality drops, the parallel conductivity rises and the mode is stabilized—this corresponds to region III. Thus the stability boundary is the boundary between regions II and III. Numerical calculation (qualitatively supported analytically) indicates that the rippling mode is unstable for

$$T_{\mathrm{e}} < \frac{3.08 \times 10^4}{(k_y L_{T\mathrm{e}})^{2/3}} \left[\frac{B^2 L_{T\mathrm{e}}}{T_{0\mathrm{e}}^{3/2}}\left(\frac{L_{\mathrm{s}}}{q_0 R}\right)\right]^{2/3} \tag{5.3.1}$$

with $k_y L_T \sim m$, the poloidal mode number. Here, the units are SI except for temperature which is measured in eV. The subscript zero indicates the parameter measured at the plasma centre, which enters because of the model chosen for the plasma current density $[j_\parallel = 2(B/q_0 R)(T/T_0)^{3/2}]$. For JET-like parameters this critical temperature is of the order of a few 10's eV and therefore the mode will only exist right at the plasma edge (if at all). The transport due to turbulence induced by the rippling mode is analysed by Garcia *et al* (1985).

As mentioned earlier, a radial variation in the impurity concentration can also give rise to a resistivity gradient and thus cause rippling-mode turbulence. Impurities are included in the analysis of Hahm *et al* (1987) where it is found that their effect is to give an additive contribution to the total transport due to resistivity-gradient driven turbulence. Whether the transport is enhanced or diminished by the impurity gradient depends on whether the impurity concentration increases or decreases towards the plasma edge. A linear calculation by Tang *et al* (1988) indicates that the critical temperature for stability (i.e. (5.3.1)) is raised if the impurity profile increases towards the plasma edge (though it is still expected to lie in the region of the order a few 10s of eV).

The most recent modification to the theory is by Thayer and Diamond (1987, 1990) who include radiation effects due to impurities, which may be important at the plasma edge. The earlier work also encompasses the previously mentioned calculations involving resistivity-gradient turbulence so we shall restrict ourselves to a description of that. It considers turbulence driven by resistivity and impurity gradients with saturation occuring because the turbulence enhances the parallel conduction of heat and impurities which damps the energy source. The effects of radiative cooling are also included and are found to enhance the transport. The model which is employed consists of four fluid equations describing a parallel Ohm's law, vorticity evolution, resistivity evolution (or, equivalently the temperature evolution) and an equation describing the dynamics of the impurities (which is derived from the continuity equation for the impurity and main species ions). Radiative cooling is due to an impurity radiation rate, $I_Z(T)$, defined such that it cools the temperature according to

$$\frac{3}{2}\frac{\mathrm{d}T}{\mathrm{d}t} \sim -n_Z I_Z(T) \tag{5.3.2}$$

where n_Z is the impurity density. The dependence of the particle diffusion coefficient on $I_Z(T)$ appears through the 'growth rates' γ_R and γ_Z where

$$\gamma_R = \frac{2}{3} n_Z \left(\frac{I_Z}{T} - \frac{\mathrm{d}I_Z}{\mathrm{d}T}\right) \tag{5.3.3}$$

$$\gamma_Z = \left(\frac{n_0}{Z^2} + n_Z\right)\frac{I_Z}{T} \tag{5.3.4}$$

and n_0 is the main species (singly charged) ion density. For a low Z impurity such that

$$n_Z Z \ll n_\mathrm{i} \qquad n_Z Z^2 \simeq n_\mathrm{i} \tag{5.3.5}$$

the following equation is derived for the particle diffusion coefficient, D_n:

$$\frac{D_n}{D_0} = \left\{1 + \eta_Z\left[1 + (\Gamma_Z - \Gamma_R)\left(\frac{D_n}{D_0}\right)^{-1/2}\right] + \Gamma_R\left(\frac{D_n}{D_0}\right)^{1/4}\right\}^{4/3} \tag{5.3.6}$$

with

$$D_0 = \left(\frac{L_\mathrm{s} J_z \eta}{L_\eta B_z}\right)^{4/3} (\chi_Z k_\parallel'^2)^{-1/3} \tag{5.3.7}$$

where η is the resistivity, J_z is the toroidal current density, B_z is the toroidal field, L_s is the shear length, $\eta_Z = L_\eta/L_Z$, $L_Z = [\mathrm{d}(\ln Z_\mathrm{eff})/\mathrm{d}r]^{-1}$ and $L_\eta = [\mathrm{d}(\ln \eta)/\mathrm{d}r]^{-1}$. The impurity parallel diffusion, $\chi_Z = (m_\mathrm{i}/m_Z)^{1/2} v_\mathrm{thi}^2/(Z^2\nu_\mathrm{ii})$ and χ_T is the thermal parallel diffusion of the electrons ($\chi_T \sim \chi_Z Z^2 (m_Z/m_\mathrm{e})^{1/2}$). The parameters Γ_R and Γ_Z are related to γ_R and γ_Z through

$$\Gamma_R = \gamma_R/\bar{\gamma}_R \qquad \Gamma_Z = \gamma_Z/\bar{\gamma}_Z \tag{5.3.8}$$

where

$$\bar{\gamma}_R = \left(\frac{L_s J_z \eta}{L_\eta B_z}\right)^{2/3} (\chi_T k_\parallel'^2)^{1/3} \qquad \bar{\gamma}_Z = \bar{\gamma}_R (\chi_Z/\chi_T)^{1/3} . \tag{5.3.9}$$

Finally, $k_\parallel = (m - nq)/(Rq) = mx/(rL_s)$ where R is the major radius, m the poloidal mode number and r is the minor radius of the rational surface about which the mode is centred. The prime on $k_\parallel$ indicates derivative with respect to the distance from the surface, x; thus $k_\parallel' = m/(rL_s)$.

Equation (5.3.6) can be solved in certain limiting cases. When radiation effects are small ($\Gamma_R \ll 1$, $\Gamma_Z \ll 1$)

$$D_n = D_0(1 + \eta_Z)^{4/3} \tag{5.3.10}$$

which is the result derived by Hahm *et al* (1987) where the resistivity-gradient driven mode was investigated without consideration of the radiation effects. The limit of no impurities (i.e. $\eta_Z \to 0$, the rippling mode) is in agreement with the calculation of Garcia *et al* (1985). Expressions are also given for the impurity, D_Z, and temperature, D_T, diffusivities:

$$D_Z = D_n \qquad D_T = (\chi_Z/\chi_T)^{1/3} D_n . \tag{5.3.11}$$

For $\Gamma_R \gg 1, \eta_Z$

$$D_n = D_0 \Gamma_R^2 \tag{5.3.12}$$

while in the limit that $\Gamma_Z \gg \Gamma_R$, $\Gamma_Z \gg 1$, $\eta_Z \Gamma_Z \gg \Gamma_R$ and $\eta_Z \Gamma_Z \gg 1$ are satisfied, we have

$$D_n = D_0(\eta_Z \Gamma_Z)^{4/5} . \tag{5.3.13}$$

The above work makes the assumption that the pressure remains constant (i.e. the density fluctuations exactly cancel the temperature fluctuations). In fact this is a poor approximation and leads to a significantly larger transport due to the thermal instability than would otherwise be obtained. This is demonstrated in a later calculation by Thayer and Diamond (1990) where the model considered consists of the evolution equations for the temperature, density and parallel velocity fluctuations (the impurity density profile is taken to be constant in this calculation, i.e. $\eta_Z = 0$). In the analysis described above the effects of the radiative instability could be described in terms of a single growth rate γ_R (see (5.3.3)) due to the fact that $\tilde{n}/n = -\tilde{T}/T$. If this constraint is dropped, then two 'growth rates' can be defined—one describing the instability driven by density fluctuations, γ_n and the other by temperature fluctuations:

$$\gamma_n = \frac{2}{3} n_Z \frac{I_Z(T)}{T} \qquad \gamma_T = -\frac{2}{3} n_Z \frac{\mathrm{d}I_Z}{\mathrm{d}T} \tag{5.3.14}$$

(where $\gamma_R = \gamma_n + \gamma_T$). The drive due to density fluctuations (called the condensation instability drive) is a robust instability because if more than one impurity exists many $I_Z(T)$ spectra contribute to a total, thus enhancing the growth. The same is not true of the thermal instability drive, however, because for a given temperature, T, $\mathrm{d}I_Z/\mathrm{d}T$ can have different signs for different impurity elements, and therefore extra impurities can either enhance or suppress the growth of the instability. For this reason, the thermal instability is also referred to as a 'fickle' instability. The calculation proceeds in a similar manner to the one described above in order to derive the factor by which radiation enhances the radial diffusion. No diffusion coefficients are calculated in this work, but the result indicates that Γ_R of the previous theory can be replaced by

$$\Gamma_R \to \Gamma_T + (\chi_n/\chi_T)^{2/3} \Gamma_n \tag{5.3.15}$$

where $\Gamma_j = \gamma_j v_{r0}^{-2/3}(\chi_j k_\parallel'^2)^{-1/3}$ for $j = T, n$, with $v_{r0} = 3V_{\rm loop}L_s/(4\pi R_0 B_0 L_T)$ and $\chi_n = v_{\rm thi}^2/\nu_{\rm iz}$, $\chi_T = v_{\rm the}^2/\nu_{\rm ee}$. For typical tokamak parameters $\chi_n/\chi_T \sim 10^{-3}$ and so the replacement of (5.3.15) represents a significant reduction in the transport. In particular, in the (radiation dominated) limit, $\Gamma_j \gg 1$,

$$D_n = D_0[\Gamma_T + (\chi_n/\chi_T)^{2/3}\Gamma_n]^2 \tag{5.3.16}$$

thus indicating that the robust condensation instability has a negligible effect on transport, which is now seen to be dominated by the fickle instability. An estimate of the level of reduction in the transport can be obtained by evaluating the ratio Γ_T/Γ_R. Assuming $\gamma_n \gtrsim \gamma_T$ (5.3.16) gives rise to at least a factor of four reduction compared to (5.3.12). Thus the diffusion due to a radiation dominated instability is actually much lower than was originally thought and (5.3.16) should be used in preference to (5.3.12).

The above analyses of the resistivity-gradient-driven mode employ the reduced, resistive MHD equations which are strictly only valid in the Pfirsch–Schlüter collisionality regime. Kwon *et al* (1989) perform a calculation of the transport due to the mode using neoclassical equations, which are relevant for a description of a plasma in the banana-plateau collisionality regimes. The principal difference between this calculation and those of reduced MHD is that the resistivity in the banana regime becomes dependent on the plasma density as well as the temperature and thus the rippling mode (which, as described earlier, is driven by a resistivity gradient) is now able to tap the free energy source of the density gradient. Impurity gradient effects, radiative cooling and ω_{*e} terms are not included and one-point renormalization theory is used to derive coupled equations for the time evolution of the density and temperature fluctuations. Requiring a stationary saturated solution provides a solubility condition on the resulting equations, from which expressions for the radial diffusivities of the temperature and density can be evaluated. These are:

$$D^t = \left[(C_n + C_t\eta_e)\left(\frac{cE_\parallel L_s}{BL_n}\right)\right]^{4/3}[\chi_\parallel k_\parallel'^2]^{-1/3} \tag{5.3.17}$$

$$D^n = \left[(C_n + C_t\eta_e)\left(\frac{cE_\parallel L_s}{BL_n}\right)\right]^{4/3}[\chi_n k_\parallel'^2]^{-1/3} \tag{5.3.18}$$

where $D^{n,t}$ are the radial diffusivities for density and temperature. The parameter $\chi_n = c_s^2/\mu_i$ where $\mu_i = 0.66\epsilon^{1/2}\nu_i/(1+1.03\nu_{*i}^{1/2}+0.31\nu_{*i})$ and $\chi_\parallel$ is the electron parallel thermal diffusivity. Finally,

$$C_n = \frac{4.51\epsilon^{1/2}(0.51\nu_{*e}^{1/2} + 1.07\nu_{*e})}{(1+1.02\nu_{*e}^{1/2} + 1.07\nu_{*e})^2}\left(1 + \frac{4.51\epsilon^{1/2}}{1+1.02\nu_{*e}^{1/2}+1.07\nu_{*e}}\right)^{-1} \tag{5.3.19}$$

$$C_t = \tfrac{3}{2} - 2C_n\,. \tag{5.3.20}$$

All the diffusivities presented in this section depend on the poloidal mode number m and a value for this needs to be selected. Assuming it to be a constant with respect to the plasma parameters, it can be absorbed into the normalization coefficient (to be fixed by comparison with experiment). Scaling arguments indicate that (for large m) m is not a constant with plasma parameters but in fact scales as

$$\langle m^2\rangle \sim \frac{\mu_0 S^{1/2}K^{-5/2}rRqJ_z}{L_\eta s^3 B} \tag{5.3.21}$$

where $K = \chi_\parallel\tau_A/(Rq)^2$ for a pure plasma and $K = \chi_Z\tau_A/(Rq)^2$ when impurities are present (Connor 1988). If (5.3.21) predicts a small value for m then this is outside its region of validity and m could be regarded as a constant to be absorbed into the normalization coefficient. (It is interesting to note that the numerical calculation of $\langle m^2\rangle^{1/2}$ in the case of the resistive ballooning mode appears to indicate saturation at $\langle m^2\rangle^{1/2} \sim 3$ (see (5.2.13)). This may represent a cascade to long wavelengths ($n \sim 1$) with $\langle m^2\rangle^{1/2}$ determined by the geometry (i.e. $n = 1$, $q = 3$ and $m = nq$) in which case one is justified in taking $\langle m^2\rangle^{1/2}$ to vary like q).

5.4. Conclusions

The fluid models used in this section are relatively simple. This has the consequence that powerful scale-invariance arguments can often determine the form of the diffusivities uniquely, up to a normalization constant. Furthermore, numerical simulation has been widely used to corroborate and guide analytic turbulence treatments. Rippling-type modes including impurity radiative cooling effects tend to be restricted in their validity to the plasma edge but detailed and encouraging comparisons between theory and experiment (e.g. Leboeuf *et al* 1991) have been carried out. While other resistive models are also valid only near the plasma edge, generalizations such as including neoclassical effects or electron inertia allow them to be extended to the core. Those involving pressure-gradient drives are promising contenders for describing L-mode power degradation. Furthermore they are able to produce thermal diffusivities which increase to the plasma periphery. These features are evidenced by the comparisons with JET data by Tibone *et al* (1994).

6. Overall conclusions

The main purpose of this review has been to provide a source of theoretical expressions for anomalous thermal transport coefficients available for ready comparison with experimental data—as a corollary validity constraints are emphasized. In addition we have sketched the theoretical models employed in deriving the expressions. We have concentrated on the more recent developments in ion and electron anomalous thermal diffusivities, extending and bringing up to date earlier reviews by Liewer (1985), Ross *et al* (1987) and Horton (1990). In section 2, after setting the scene by describing earlier work, we considered the extensive recent literature on slab η_i, toroidal ∇T_i and trapped-ion modes. The latter two modes are often considered as serious contenders as an explanation for χ_i and detailed calculations of the stability criteria have been carried out. However, most of the resulting expressions for χ_i are of the gyro-Bohm-type and fail to describe the experimental profile, though confinement time scalings with respect to power, density and, for some more recent models, current are reasonable. Considerable effort continues in an attempt to produce Bohm-like features and better radial profiles.

Section 3 addresses the many drift-wave models for electron transport, some generic only depending on the existence of drift-wave turbulence, others related to specific instabilities. The large number of models stems from the many instabilities, choices of mode structure and saturation mechanisms and whether the fluctuations are electrostatic or electromagnetic. Again the models are invariably gyro-Bohm and have difficulty with radial profiles even when they are able to reproduce confinement time scalings.

Theories of electron transport based on nonlinear magnetic island instabilities have been very popular recently, with many mechanisms, such as bootstrap current or FLR effects, being invoked to cause their growth. As a result these models are described in somewhat more detail in section 4. Island growth mechanisms tend to increase with β_p suggesting a link with power degradation observed in L-mode. However, the most successful model, the semi-empirical gyro-Bohm Rebut–Lallia–Watkins model, lacks a real theoretical derivation. Transport due to stochastic magnetic fields caused by these islands according to the Rechester–Rosenbluth formula tends not to depend on the actual driving mechanism. However, there remains a freedom in the choice of the number of contributing islands which can affect the ρ_* scaling of χ_e. The somewhat heuristic ion pumping model of Kadomtsev is independent of ρ_* but captures features of the experimental χ_e for L-modes.

Finally in section 5 resistive fluid models are described. These simple models invite the use of scale invariance and numerical simulation to support analytic treatments. The neoclassical and electron inertia modified versions of the pressure-gradient models can be extended into the core and have a number of encouraging features for describing L-mode plasmas.

A number of the transport coefficients given in this review rely on the Rechester and Rosenbluth theory for transport in a stochastic magnetic field which refers to test-particle transport. Many authors assume this to be equal to the electron heat transport. However, Terry *et al* (1986) question this association because it neglects important self-consistent field effects; when these are taken into account the transport is governed not by the magnetic fluctuations but by the *electrostatic* fluctuations. Later work by Krommes and Kim (1988) criticises the analysis of Terry *et al* and claims that such estimates of transport are often adequate.

Comparison between many of the theories considered in this review and JET data have been carried out in Connor *et al* (1993) and Tibone *et al* (1994). The reader is invited to refer to these papers for details but the general picture is

(i) the χ_i models are generally unsuccessful and refinements are needed
(ii) some electron models, in particular stochastic transport due to electromagnetic drift waves, Kadomtsev's ion pumping of magnetic islands and pressure-gradient fluid models with electron inertia included in the Ohm's law, are more promising.

Much recent theoretical activity, not considered in this review, has been devoted to the role of the radial electric field or sheared toroidal and poloidal flows on stability and transport, since these are thought to be involved in the confinement improvement of H-modes. Another topical theme is seeking explanations for global mode structures (e.g. due to toroidal coupling or cascades to long wavelength) in an attempt to justify Bohm-like scalings.

As a final remark we comment on the many expressions for diffusivities described in this review. These result from a multitude of limits and assumptions employed in the analyses and could be reduced in number if more numerical simulation were employed to isolate which of these results are really justified. Thus considerable progress with transport due to the slab η_i mode has resulted from confronting analytic estimates with numerical simulation results. Furthermore, those features of the theories which are really robust should be emphasized rather than factors 'teased' out of the analysis in some extreme limit or from an assumed model for the saturation mechanism.

We conclude this review with the hope that it provides a useful reference for those wishing to interpret transport data in terms of theoretical models and a basis for future reviews.

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Appendix A. Standard definitions

In this appendix we list the standard definitions which we use in this review.

k_B Boltzmann constant R major radius η resistivity
ρ_m mass density

$$c_s = \sqrt{\frac{k_B T_e}{m_i}} \qquad v_{\mathrm{th}\,j} = \sqrt{\frac{2k_B T_j}{m_j}} \qquad \tau = \frac{T_e}{T_i}$$

$$\Omega_j = \frac{e_j B}{m_j} \qquad \rho_j = \frac{v_{\mathrm{th}\,j}}{\Omega_j} \qquad \rho_s = \frac{c_s}{\Omega_i}$$

$$L_{nj}^{-1} = |\nabla \ln n_j| \qquad L_{Tj}^{-1} = |\nabla \ln T_j| \qquad L_s = \frac{Rq}{s}$$

$$\beta = \frac{2\mu_0 \int p\,\mathrm{d}V}{\int B^2 \mathrm{d}V} \qquad \beta_p = \frac{2\mu_0 p}{B_\theta^2} \qquad \eta_j = \frac{L_{nj}}{L_{Tj}}$$

$$\epsilon_{nj} = \frac{L_{nj}}{R} \qquad \epsilon_{Tj} = \frac{L_{Tj}}{R} \qquad \nu_{*j} = \frac{\nu_j}{\epsilon \omega_{bj}}$$

$$\omega_{bj} = \frac{\epsilon^{1/2} v_{\mathrm{th}\,j}}{Rq} \qquad \omega_{tj} = \frac{v_{\mathrm{th}\,j}}{Rq} \qquad \omega_{dj} = \frac{k_\theta \rho_j v_{\mathrm{th}\,j}}{R}$$

$$\omega_{*j} = \frac{k_\theta \rho_j v_{\mathrm{th}\,j}}{2L_n} \qquad q = \frac{1}{2\pi} \oint \frac{r B_\phi}{R B_\theta} \mathrm{d}\theta \qquad s = \frac{r}{q}\frac{\mathrm{d}q}{\mathrm{d}r}$$

$$\omega_{pe} = \left(\frac{n_e e^2}{m_e \epsilon_0}\right)^{1/2} \qquad \tau_R = \frac{\mu_0 r^2}{\eta} \qquad \tau_A = \frac{(\mu_0 \rho_m)^{1/2} Rq}{B}$$

$$S = \frac{\tau_R}{\tau_A}$$

Appendix B. Summary of diffusivities

The following pages provide summary tables of the various expressions for the transport coefficients that have been described in the text. Standard definitions are given in appendix A. However, in order to avoid a lengthy list of the non-standard definitions, the reader is refered to the relevant section of the main text for these. We have used the same notation for these tables as in the main text and the units are SI unless specifically stated otherwise. In the table for transport induced by ITG turbulence we shall usually quote results for F, where $\chi_i = F\rho_s^2 c_s/L_n$. In the table for magnetic island theories we have defined

$$\chi_e^{(1)} \sim \frac{v_{\mathrm{th}\,e} w^3 m}{L_s r} \qquad \chi_e^{(2)} \sim \epsilon^2 \frac{v_{\mathrm{th}\,e} R}{q^3 s^2} \left(\frac{\alpha_s}{N}\right)^3 .$$

Table A1. Thermal diffusivities for ITG modes.

Author	Comments	F	Validity criteria
Mattor (1989)	Mixing-length calculation; interpolation of small and large η_i limits; threshold is weak/strong turbulence boundary	$F = 0.04\left(\frac{L_s}{L_n}\right)^2\left(\frac{\tau}{1+\tau}\right)^{3/2}(\eta_i - \eta_{ic})^3$	Slab $L_s < R/2$ $\eta_i > \eta_{ic}$, $\eta_{ic} = 1.2$
Hamaguchi (1990)	3D fluid simulation	$F = \frac{1}{\tau}(\eta_i - \eta_{ic})\exp\left(-5\frac{L_n}{L_s}\right)$	Slab $L_s < R/2$ $\eta_i > \eta_{ic}$
Hassam (1990)	Long-wavelength modes, $k_\parallel v_{thi} < \nu_{ii}$. The scaling with η_i is from a figure in this work.	$\begin{cases} F \sim \frac{L_s^2}{\tau L_n^2}\left(\frac{\nu_{ii} L_n}{c_s}\right)(\eta_i - 2/3)^2 \\ F \sim \tau^{-3/2}(\eta_i - 0.9)^{3/2} \end{cases}$	$\begin{cases} \eta_{ic} = 2/3 \\ \eta_{ic} = 0.9 \end{cases}$ Slab $L_s < R/2$ $\eta_i > \eta_{ic}$
Biglari (1989)	Uses fluid equations. Mixing-length calculation	$F = \left(\frac{q}{s}\right)\frac{(1+\eta_i)^{1/2}}{\tau}$ Also gives	Toroidal, $L_s > R/2$ $\eta_i \gg 1$
		$D_e \sim \chi_e \sim \epsilon^{3/2}\frac{\rho_s^2 c_s^2}{L_n^{3/2} R^{1/2} \nu_e}\left(\frac{1+\eta_i}{\tau}\right)^{3/2}\frac{q}{s}\left(\frac{1}{1+0.1/\nu_{*e}}\right)$	$\nu_{*e} < 1$
Guo (1989)	Fluid limit of gyrokinetic theory.	$F = 0.6\epsilon_n\left[\frac{(1+2q^2)^3}{sq(\tau\epsilon_T)^3}\right]^{1/4}$	Toroidal, $L_s > R/2$ $\epsilon_T \ll 1$, $\eta_i \gg \eta_{ic}$
Hong (1990)	Solution of 2D fluid equations.	$F = 2\frac{\epsilon_n}{\tau^{1/2} s}(\eta_i - 2/3)^{1/2}$	Toroidal, $L_s > R/2$ $\rho_s/L_n < s < 2\epsilon_n$
	Low shear calculation.	$\chi_i \sim \tau^{3/4}\epsilon_n^{3/4}(\eta_i - \eta_{ic})^{1/4}\rho_s c_s$	Toroidal, $L_s > R/2$ $\rho_s/L_n > s$

Table A1. (Continued).

Author	Comments	F	Validity criteria
Dominguez (1989)	Modified mixing-length calculation. Fluid.	$F = 3.53\frac{q}{s}\eta_i\sqrt{1-\frac{L_{Ti}}{L_{Ti}^c}}\Theta\left(1-\frac{L_{Ti}}{L_{Ti}^c}\right)$	Toroidal, $L_s > R/2$ Flat density profile, $\epsilon_n \gg 1/2$ $\frac{1}{L_{Ti}^c} = \left[\frac{5}{R}, \frac{1}{L_n}\right]_{\max}$
Hong (1986)	Gyrokinetic theory	$F = 2\frac{q}{s}\left(\frac{1+\eta_i}{\tau}\right)k_0\left[1-\left(\frac{1+\eta_i}{2\gamma_0\tau}\right)^2 k_0^4\right]^{-1}$	Toroidal, $L_s > R/2$ $\epsilon_n < \tau/2$, $\eta_i \gtrsim 2$ $(s/q)^{1/2}[(1+\eta_i)/\epsilon_n]^{1/4} \ll 1$
Romanelli (1989)	Quasilinear mixing-length calculation. Kinetic. Valid for all η_i above threshold.	$F = 14.1\frac{\epsilon_n^{1/2}}{\tau^{3/2}}(\eta_i - \eta_{ic})^{1/2}$	Toroidal, $L_s > R/2$ $\epsilon_T \ll 1$ $\eta_i > \eta_{ic}$ $\eta_{ic} = \begin{cases} 1 & \epsilon_n < 0.2 \\ 1 + 2.5(\epsilon_n - 0.2) & \epsilon_n > 0.2 \end{cases}$
Romanelli (1991)	Long-wavelength ITG mode—'ion toroidal mode'	$F = \frac{1}{\sqrt{2}\tau^{3/2}}\frac{q\eta_i}{s^2}$	Toroidal, $L_s > R/2$ $\lambda \gg 1$ $\lambda > \frac{1}{2}\left(\frac{2}{\tau}\frac{1+\tau^{-1}}{1-(2/\eta_i)}\right)^{1/2}$
	'Slab mode'	$F = 0.5\frac{q}{\tau}\left(\frac{\eta_i\epsilon_n}{s}\right)^{1/2}$	Toroidal, $L_s > R/2$ $7\sqrt{\pi}(\epsilon_T s\tau)^2 \ll \lambda \ll [7\sqrt{\pi}(\epsilon_T s\tau)^2]^{-1/4}$ $\eta_i \gg \left(\frac{32}{7\sqrt{\pi}}\right)\frac{\tau^{3/5}\lambda^{1/5}}{(\epsilon_T s)^{2/5}}$
Kim (1991)	Neoclassical fluid theory	$\chi_i < 2.3\mu_i\rho_s^2\frac{q^2}{\epsilon^2}(1+\eta_i)$	Toroidal, $L_s > R/2$ $\eta_i \gg \eta_{ic}$ $\sqrt{2}\tau^{1/2}\epsilon^{1/2}\epsilon_n^{1/2} \gg qk_\theta\rho_s$ $\nu_* \ll \epsilon^{-3/2}$

Table A1. (Continued).

Author	Comments	F	Validity criteria
Biglari (1988)	Ion-pressure-gradient driven trapped-ion mode. Clump theory.	$F \simeq \frac{1}{\sqrt{2}}\left(\frac{m_e}{m_i}\right)^{1/2}\frac{q}{\epsilon_n \nu_{*e}}\left(\frac{\eta_i}{s\tau}\right)^2\left(\frac{\eta_i}{\eta_{ic}}-1\right)$	$\nu_{*e,i} \ll 1$ $\epsilon_n \ll 1$ $k_\theta \rho_s \ll \epsilon_n \frac{(2\epsilon\tau)^{1/2}}{q}$ $\eta_i > \eta_{ic} = (1.5 - \bar{E})^{-1}$ $\nu_{*e} \gg \frac{(k_\theta \rho_s) q \eta_i}{\sqrt{2}\epsilon^{1/4} s \epsilon_n \tau}\left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{\eta_i}{\eta_{ic}}-1\right)^{1/2}$
Biglari (1989)	Ion-pressure-gradient driven trapped-ion mode. Two cases considered: collisionless and collisional (via pitch angle scattering).		
	Collisionless case	$F \sim 2^{3/4}\frac{\epsilon^{1/4}\epsilon_n^{1/2}}{(k_\theta \rho_s)s^2}\left(1+\frac{\eta_i}{\tau}\right)^{1/2}$	$\epsilon_n \ll 1/2$ $k_\theta \rho_s \ll (2\tau\epsilon)^{1/2}\epsilon_n/q$ $\nu_{*i} \ll 1$ Mode unstable for: $2(1+\eta_i) > \frac{\epsilon^{1/2}\tau^2}{\sqrt{2}(1+\tau)\epsilon_n}$
	Collisional	$F = 2\sqrt{2}\frac{q\tau^{1/2}\eta_i}{s^2 \nu_{*i}\epsilon^{1/4}}$	$\epsilon_n \ll 1/2$ $k_\theta \rho_s \ll (2\tau\epsilon)^{1/2}\epsilon_n/q$ Mode unstable for: $\eta_i > 4/3$ and $\nu_{*i} < \epsilon^{-1/4}\epsilon_n^{-1/2}(k_\theta \rho_s) q \tau^{1/2}$

Table A1. (Continued).

Author	Comments	F	Validity criteria
Xu (1990)	Electrostatic and purely magnetic trapped-ion modes. Gyrokinetic approach.	Electrostatic trapped mode (collisionless): $F = 2\sqrt{2}\frac{\epsilon^{3/4}\epsilon_n^{1/2}a_1^{3/2}\eta_i^{1/2}q}{[1+h^2(\theta)]\tau^{3/2}\nu_{*i}}$	$\nu_{*i} \ll a_1\frac{\sqrt{2}q(k_\theta\rho_s)}{\tau^{1/2}\epsilon^{1/2}}$ $\tau^{1/2}\epsilon^{1/2}\nu_{*i} \ll \sqrt{2}k_\theta\rho_s q \ll \epsilon^{3/4}\tau^{1/2}$ Mode unstable for: $\alpha < \frac{1.71+s}{1+7/(6q^2)}$
		Electrostatic trapped mode (collisional): $F = 2\sqrt{2}\left(\frac{\epsilon}{\tau}\right)^{3/2}\frac{q\eta_i^2}{[1+h^2(\theta)]\nu_{*i}\epsilon_n}$	$\nu_{*i} \gg a_1\frac{\sqrt{2}q(k_\theta\rho_s)}{\tau^{1/2}\epsilon^{1/2}}$ $k_\theta\rho_s \gg \frac{\epsilon_n\nu_{*i}\tau^{1/2}}{\sqrt{2}\eta_i q}$
		Pure magnetic trapped mode: $F \sim 2\sqrt{2}a_1\frac{q\beta(1+3\eta_i)}{\nu_{*i}\tau^{3/2}[1+h^2(\theta)]}$	$k_\theta\rho_s \gg \sqrt{2}\frac{\epsilon_n\epsilon^{1/2}\tau^{1/2}\nu_{*i}}{q}$ Mode unstable for: $\nu_{*i} < 5.7a_1^{1/2}[\beta(1+3\eta_i)]^{1/2}\frac{(k_\theta\rho_s)q}{\epsilon^{1/4}\tau^{1/2}\epsilon_n^{1/2}}$
Diamond (1990)	Dissipative trapped-ion convective cell turbulence	$F = \frac{3}{2\sqrt{2}}\frac{\epsilon^{1/2}}{s^2}\left(\frac{m_e}{m_i}\right)^{1/2}\frac{q}{\epsilon_n\nu_{*e}}$ also $\chi_e \simeq 3D/2 \simeq \chi_i$	$\nu_{*e} \ll \sqrt{2}sk_\theta^2\left(\frac{m_e}{m_i}\right)^{1/2}\frac{\rho_s\eta_i Rq}{\epsilon^{1/2}}$ $\tau = 1$

Table A2. Thermal diffusivities for electron drift waves.

Author	Comments	χ_e	Conditions
Hirshman (1979)	Collisionless slab electron drift wave. Effects of electrostatic turbulence	$\chi_e \sim \frac{45}{2}\Delta_{PB}^{3/2}\tau^{5/2}\left[\frac{1+\tau}{2}\right]^{-9/2}\frac{c_s\rho_s^2}{L_s}$	Complicated stability criterion could be obtained from reference. $\nu_e < \omega \sim \omega_{*e}$
Molvig (1979)	Collisionless slab electron drift wave. Effects of electromagnetic turbulence	$\chi_e \sim \frac{0.07}{\tau^{3/2}}\left(\frac{\tau}{1+\tau}\right)^4\left(\frac{L_s}{L_n}\right)^2\frac{c_s}{L_n}\left(\frac{c}{\omega_{pe}}\right)^2$	$\beta_e > (m_e/m_i)$ $\nu_e < \omega \sim \omega_{*e}$
Waltz (1986)	$\gamma/k_\perp^2$ estimate. Collisionless slab electron drift wave.	$\chi_e \sim \left(\frac{m_e}{m_i}\right)^{1/2}\frac{q}{\epsilon_n}\frac{\rho_s^2 c_s}{L_n}$	$\nu_e < \omega_{te}$
	Collisional mode	$\chi_e \sim \left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{\nu_e}{\omega_{te}}\right)\frac{q}{\epsilon_n}\frac{\rho_s^2 c_s}{L_n}$	$\nu_e > \omega_{te}$
Tang (1986)	Profile consistency method assuming the trapped-electron mode to be dominant in the confinement zone	Ohmic heating alone: $\chi_e = \chi_{e0}F(r)$ Ohmic + auxiliary heating: $\chi_e = \chi_{eh}F_h(r)$	$\nu_{*e} < 1$ between the $q = 1$ and $q = 2$ surfaces.
Dominguez (1987)	$\gamma/k_\perp^2$ estimate of diffusion due to circulating/trapped-electron modes and the η_i mode.	$\chi_e = \frac{5}{2}(c_{te}\hat{D}_{te} + c_{ce}\hat{D}_{ce})(1 + \bar{c}_{ei}f_{i,\,th}\epsilon_n\eta_i)$ $\chi_i = \frac{5}{2}[c_i\hat{D}_i f_{i,\,th} + \bar{c}_{ie}(c_{te}\hat{D}_{te} + c_{ce}\hat{D}_{ce})]$	Transitions between collisional and collisionless modes are governed by a switch which is incorporated in the $\hat{D}$
Perkins (1984)	Mixing-length calculation using random step size arguments. Collisional trapped-electron mode.	$\chi_e \sim \epsilon^{1/2}\frac{\omega_{*e}}{k_\perp^2}\left(\frac{\omega_{*e}}{\nu_{eff}}\right)$	$\nu_{*e} > \omega_{*e}/\omega_{be}$
	Collisionless trapped-electron mode	$\chi_e \sim \epsilon^{1/2}\frac{\omega_{*e}}{k_\perp^2}$	$\nu_{*e} < \omega_{*e}/\omega_{be}$

Table A2. (Continued).

Author	Comments	χ_e	Conditions
Romanelli (1986)	$\gamma/k_\perp^2$ estimate of transport due to trapped-electron mode.	$\frac{5}{2\sqrt{2}}\frac{q\eta_e}{\epsilon_n\nu_{*e}}\left(\frac{m_e}{m_i}\right)^{1/2}\frac{\rho_s^2 c_s}{L_n}\frac{1}{1+0.1/\nu_{*e}}$	Transition from collisional to collisionless mode is contained within the expression for χ_e
Gang (1991)	Weak turbulence calculation of trapped electron induced transport. Sheared slab geometry. Kinetic treatment.	$\chi_e = 0.8\tau^{-3/2}\left(\frac{L_n}{L_s}\right)^{1/2}\left(A_e^l\frac{\bar{\gamma}}{s}\right)^2 H(\bar{k}_\theta^c)\frac{\rho_s^2 c_s}{L_n}$ $\chi_i = 0.8\tau^{-3/2}\left(\frac{L_n}{L_s}\right)^{1/2}\left(A_e^l\frac{\bar{\gamma}}{s}\right)^2 F(\bar{k}_\theta^c)\frac{\rho_s^2 c_s}{L_n}$ $D = 0.4\tau^{-3/2}\left(\frac{L_n}{L_s}\right)^{1/2}\left(A_e^l\frac{\bar{\gamma}}{s}\right)^2 F(\bar{k}_\theta^c)\frac{\rho_s^2 c_s}{L_n}$	Two collisionality regimes: Collisionless $\omega_{*e} > \omega_{de} > \nu_{eff}$ Collisional $\nu_{eff} > \omega_{*e} > \omega_{de}$ The definitions of the various parameters differ for the different parameter regimes. Must satisfy $\nu_{*e} < 1$ in both regimes. $\eta_i, \eta_e \ll 1$
Similon (1984)	Strong turbulence theory. Toroidicity induced mode structure. Electrons treated linearly.	Circulating electron mode: $\chi_e \sim 0.2\left(\frac{m_e}{m_i}\right)^{2/3}\left(\frac{q}{\epsilon_n}\right)^{4/3}\frac{1}{s^{7/3}\nu_{*e}^{1/3}}\frac{\rho_s^2 c_s}{L_n}$ Collisional mode: $\chi_e \sim 0.2\left(\frac{m_e}{m_i}\right)^{2/3}\left(\frac{q}{\epsilon_n}\right)^{4/3}\frac{1}{s^{7/3}\nu_{*e}^{4/3}}\frac{\rho_s^2 c_s}{L_n}$ Collisionless mode: $\chi_e \sim \frac{(2\epsilon)^{1/2}}{2\pi^2}\frac{1}{s^2\epsilon_n}\frac{\rho_s^2 c_s}{L_n}$	Circular, concentric flux surfaces. $\nu_{eff} < \omega_{be}$ $\nu_{eff} > \omega_{de}$ $\nu_{eff} < \omega_{de}$
Hahm (1991)	Weak turbulence theory of the collisionless trapped electron mode. Treats both ions and trapped electrons in a nonlinear way. Toroidal mode structure.	Complicated expressions for the particle diffusion coefficient, D and the thermal diffusivities, $\chi_{e,i}$ given by equations (3.3.41)–(3.3.43)	$\omega_{*e} > \omega_{de} > \nu_{eff}$ $\omega_{bi} < \omega_{*e} < \omega_{be}$
Rogister (1989)	Weak turbulence calculation of heat flux due to trapped-electron mode. High transport may force $\Gamma^* = 1$.	$\chi_e \sim \frac{\rho_s^2 c_s}{L_n}\frac{L_{Te}}{L_s}\left(\frac{L_n}{L_s}\right)^2 \tau^{-1}\Gamma^* G(\Gamma^* - 1)$	$\nu_{*e} < 1$

Table A2. (Continued).

Author	Comments	χ_e	Conditions
Diamond (1983)	Clump theory of trapped-electron mode turbulence.	$\chi_e \sim \frac{\gamma_L}{k_\perp^2[1 - C(k, \omega_k)]}$	$\nu_{*e} < 1$
Kaw (1982)	Calculation of trapped-electron induced transport using transport stabilization techniques.	$\chi_e \sim \frac{\epsilon^{1/2}}{k_\perp^2} \frac{\omega_{*e}}{(1 + k_\perp^2 \rho_s^2)^2} \frac{L_s}{L_{Te}}$	$\nu_{*e} < 1$
Diamond (1990)	Short wavelength electron temperature gradient driven ('ubiquitous') mode	$\chi_e = \frac{2G}{\tau F} \frac{\epsilon_n}{s^2(k_\theta \rho_s)} \frac{\rho_s^2 c_s}{L_n}$	$\nu_{eff} < \omega_{de}$, $\nu_{*e} < 1$ $1 < k_\theta \rho_s < \frac{\epsilon^{1/2}\epsilon_n}{q}\left(\frac{m_i}{m_e}\right)^{1/2}$ Instability criterion given by equation (3.3.63).
Mikhailovskii (1976)	Short wavelength dissipative trapped-electron mode. Linear analysis.	$\chi_e \sim \frac{5.2(\bar{\nu}/\omega)^{1/2}(I_1 + \eta_e I_2)}{\tau\{\ln[32(\omega/(2\bar{\nu}))^{1/2}]\}^{3/2}} \frac{\epsilon_n}{s^2(k_\theta \rho_s)} \frac{\rho_s^2 c_s}{L_n}$ where $\bar{\nu} = \frac{3\sqrt{\pi}}{4}\nu_{eff}$ $\quad \omega = 0.58\frac{2\epsilon^{1/2}\omega_{*i}}{1 + \tau^{-1}}$	$1 < k_\theta \rho_s < \frac{\epsilon^{1/2}\epsilon_n}{q}\left(\frac{m_i}{m_e}\right)^{1/2}$ No bounds on the collisionality are given. Instability criterion: $\eta_e > 1.52$
Horton (1987) Kim (1990)	Stochastic electromagnetic turbulence. Effects of collisionality included by Kim. Note, a shear parametrization of the $\hat{D}_m$ is given in 3.2.10	$\chi_e = \epsilon^{1/2}\left\{\rho_s^2 \frac{c_s}{L_n}\hat{D}_1 + \left(\frac{c}{\omega_{pe}}\right)^2 \omega_{be}\hat{D}_2\right\}$ $+\nu_{ei}\left(\frac{c}{\omega_{pe}}\right)^2 \hat{D}_3$	Assumes that an instability exists to drive the turbulence.
Parail (1980)	The form for the thermal conductivity depends on whether the perturbation in the 'scalar potential is odd or even about the resonant surface. No driving mechanism for the turbulence is considered.	$\chi_e \sim \left(\frac{c}{\omega_{pe}}\right)^2 \frac{v_{the} s}{qR}$ $\chi_e \sim \left(\frac{r}{R}\right)^2 \left(\frac{c}{\omega_{pe}}\right)^2 \frac{v_{the}}{qR}$	Odd Even

Table A2. (Continued).

Author	Comments	χ_e	Conditions
Horton (1988)	Consider the η_e mode as a possible driving mechanism for em turbulence. Hydrodynamic treatment.	$\chi_e \sim \left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{2q}{s}\right)\eta_e^{1/2}\frac{\rho_s^2 c_s}{\tau L_n}$	Toroidal geometry. ($s < 2q$) $s/q < 1/(2\epsilon_{T_e})^{1/2}$ $\eta_e > \eta_{e,c}$ $\eta_{e,c} = \begin{cases} 2/3 & k_\perp \rho_{ei} \lesssim 0.3 \\ 1 & k_\perp \rho_{ei} \sim 0.5 \end{cases}$ $k_\parallel v_{the} \gg \omega$, $\epsilon_n < 1$
		$\chi_e \sim \left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{2q}{s}\right)(2\epsilon_{T_e})^{1/2}\frac{\rho_s^2 c_s}{\tau L_{T_e}}$	As above but $\epsilon_n \gg 1$
		$\chi_e \sim \left(\frac{c}{\omega_{pe}}\right)\frac{c_s \rho_s}{L_{T_e}} + \epsilon^{1/2}\left(\frac{c}{\omega_{pe}}\right)^2 \omega_{be}$	As above, but $k_\parallel v_{the} \ll \omega$ No bounds on ϵ_n given.
Lee (1987)	Kinetic description of η_e mode. χ_e obtained using quasilinear theory.	$\chi_e \sim 0.13\left(\frac{c}{\omega_{pe}}\right)^2 \frac{v_{the} s}{qR}\eta_e(1+\eta_e)$	Slab model. $\eta_e(1+\eta_e) \ll 69/\tau$ $\eta_e > \eta_{e,c}$ $\eta_{e,c} \sim 1$
Rozhanski (1981)	First reference to propose η_e mode as the source for the em turbulence.	$\chi_e = \epsilon\left(\frac{c}{\omega_{pe}}\right)^2 \frac{v_{the}}{a}$	Sheared slab $\beta \ll \epsilon^2$
		$\chi_e = \left(\frac{c}{\omega_{pe}}\right)\frac{c_s \rho_s}{a}$	As above, but with $\beta \gg \epsilon^2$

Table A2. (Continued).

Author	Comments	χ_e	Conditions
Zhang (1988)	No driving mechanism for the em turbulence is given. Timescale for turbulence is taken to be mode frequency (rather than $k_\parallel v_{\mathrm{th\,e}}$). Parameters ξ and α to be fitted.	$\chi_e = \xi \left(\frac{c}{\omega_{\mathrm{pe}}}\right) \rho_s c_s \left(\frac{1}{L_{T_e}} + \frac{b}{L_n}\right)$	Driving instability exists
Garbet (1990a,b)	Nonlinear collisionless microtearing turbulence. Coefficient given for $q \sim 2$, $\eta_e \sim 2$.	$\chi_e \lesssim 0.05 \frac{\rho_i^2 v_{\mathrm{th\,e}}}{L_s}$	$\nu_{*e} \ll 1$

Table A3. Diffusivities for magnetic-island theories.

Author	Comments	χ_e	Conditions
White (1989)	Current perturbation carried by particles diffusing across stochastic region	$\chi_e^{(2)}$	$\alpha_s \sim 1$
Rebut (1988) Taroni (1990)	Semi-empirical result, assuming magnetic islands exist and that a critical temperature gradient must be reached for transport.	Electron heat flux: $F_e = n_e \chi_e \nabla(kT_e)\left[1 - \left\lvert\frac{(\nabla T_e)_c}{\nabla T_e}\right\rvert\right]$ $\chi_e = 0.15\left[\frac{1}{L_{Te}} + \frac{2}{L_{ne}}\right]\frac{\tau^{1/2}}{\epsilon}\left(\frac{q^2}{q' B R^{1/2}}\right)c^2(\mu_0 m_i)^{1/2}$ Ion heat flux: $F_i = n_i \chi_i \nabla(kT_i)\left[1 - \left\lvert\frac{(\nabla T_e)_c}{\nabla T_e}\right\rvert\right]$ $\chi_i = \frac{\chi_e Z_i \tau^{1/2}}{\sqrt{1+Z_{eff}}}$ Particle diffusivity: $D \propto \chi_e\left[1 - \left\lvert\frac{(\nabla T_e)_c}{\nabla T_e}\right\rvert\right]$	$\nabla T_e > (\nabla T_e)_c$
Carrera (1986)	Bootstrap-current driven islands.	$\chi_e^{(2)}$ or $\chi_e^{(1)}$ with $w = -2.3\frac{(1-2.1\epsilon^{1/2})}{mq}\epsilon^{5/2}\beta_p\frac{RL_s}{L_n}$	$\nu_{*e} < 1 < w\nu_{*e}/r$ $w(m_{max}) = \rho_e$ $m \ll m_{max}$ $L_n < 0$ $\epsilon \ll 1$
Smolyakov (1989,1992)	Magnetic islands driven by drift effects. Evaluated for $\tau \sim 1$.	$\chi_e^{(2)}$ or $\chi_e^{(1)}$ with $w = \frac{0.25r}{m}\frac{L_s^2}{L_n^2}\frac{\eta_e}{\eta_{cr}}\left[2 - \frac{\eta_e}{\eta_{cr}}\right]\beta$	$w(m_{min}) = \rho_i$ $m \gg m_{min}$ $\eta_{cr} \approx 0.49$ $\eta_e > 2\eta_{cr}$ or $\eta_e < 0$

Table A3. (Continued).

Author	Comments	χ_e	Conditions
		$\chi_e^{(2)}$ or $\chi_e^{(1)}$ with $w^3 = -1.1 \frac{r\rho_i^2}{m} \frac{L_s^2}{L_n^2} \left[1 - \frac{\eta_e}{\eta_{cr}}\right] \left[2 + \eta_e - \frac{\eta_e}{\eta_{cr}}\right] \beta$	$w(m_{max}) = \rho_i$ $m \ll m_{max}$ $\eta_{cr} \approx 0.49$ $\eta_{cr} < \eta_e < 2\eta_e/(1 - \eta_{cr})$
Kuvshinov (1989)	Resistive-ballooning-mode driven islands	$\chi_e^{(2)}$ or $\chi_e^{(1)}$ with $w = -12.5 \frac{r}{m} \left(\frac{1}{\epsilon s}\right)^{1/2} \tan\left(\frac{\epsilon \beta_p r}{8 L_p s^{1/2}}\right)$	$L_p < 0$ $\nu_{*e} \ll 1$
Rebut (1984)	Thermal-instability-driven island	$\chi_e^{(2)}$ or $\chi_e^{(1)}$ with $w = \frac{mr}{M} \frac{\sqrt{R^2 q^2 + r^2}}{Rq}$	$P_v < 0$ $T_e \, dP_v/dT_e < P_v/2$ $w \ll r$
Kadomtsev (1991)	Considers magnetic islands driven by ions and damped by electrons.	Weak island contact: $\chi_e \sim \epsilon^{3/2} \omega_{te} \left(\frac{c}{\omega_{pe}}\right)^2$ Strong island overlap: $\chi_e \sim \frac{v_{thi}}{R^2} r^3 \beta_\theta$	$\nu_{*e} \ll 1$ Probably relevant for Ohmic plasmas $\nu_{*e} \ll 1$ Probably relevant for L,H modes
Smolyakov (1993)	Collisionless skin-depth drift magnetic islands	$\chi_e \simeq \left(\frac{c}{\omega_{pe}}\right)^2 \frac{v_{the}}{L_s} \frac{1 + \eta_e/2}{\eta_e}$	$\nu_{*e} \ll 1$

Table A4. Diffusivities for resistive fluid turbulence.

Author	Comments	D, χ_e	Conditions
Connor, Taylor (1984)	Resistive-ballooning mode. Scale invariance techniques	Mixing diffusion: $D = g_0 \frac{\eta}{\mu_0}\left(\frac{\alpha}{s}\right)$ Collisional: $\chi_e = g_1 \frac{v_{the}^2}{\nu_e}\frac{\epsilon^2}{q^2 S}\left(\frac{\alpha}{s}\right)^{5/2}$ Collisionless: $\chi_e = g_2 \frac{R\epsilon^2 v_{the}}{qS}\left(\frac{\alpha}{s}\right)^{3/2}$	$\beta q^2/\epsilon < 1$ Collisionality boundary: $\nu_e = (g_1/g_2)\omega_{te}(s/\alpha)$
Carreras (1987)	'Mixing' diffusivity. We have modified their interchange result to predict the expected behaviour of the ballooning mode. Renormalized turbulence theory. Values for m given in 5.2.13.	D as in Connor, Taylor but: $g_0 \to \frac{2}{3\pi} g_0 \ln\left(\frac{64r^4}{m^4 q^2 \tau_A^2}\left[\frac{\mu_0^2 s^2}{g_0^2 \eta^2 \alpha}\right]\right)$	Same as Connor, Taylor
Carreras (1989)	Electron heat diffusivity due to 'parallel' diffusion. Resistive-ballooning modes in collisionless limit. Suggest absorbing toroidal mode number n dependence into normalization.	$\chi_e \sim \frac{1}{2^{13/6}}\frac{1}{(n^2)^{1/3}}\frac{1}{S^{2/3}}\frac{q^2}{s}\left(\beta\frac{R}{L_p}\right)^{4/3}\frac{v_{the}r^2}{R}$	Same as Connor, Taylor Collisionless regime
Carreras (1982)	Resistive-ballooning modification of χ_e due to diamagnetic effects	See equation (5.2.15)	
Hahm (1987)	Resistive-ballooning modification due to shaped plasma cross-section (double null)	See equation (5.2.19) for modification factor	

Table A4. (Continued).

Author	Comments	D, χ_e	Conditions
Kwon (1990)	Resistive-ballooning modes using neo-classical MHD. $v_\parallel \sim v_{the}$ Absorb m into normalization	'Mixing' pressure diffusivity: $D_p = \frac{1}{8\pi}\frac{\epsilon}{q}\beta_p \frac{\eta}{\mu_0}\delta_e \frac{L_s}{L_p}\Lambda_n^2$ Parallel electron thermal diffusivity: $\chi_e = 4.6 \times 10^{-2} v_{the} L_s \left[\frac{\epsilon}{q}\beta_p\right]^{4/3} \times \delta_e^{5/3} \left[\frac{r^2}{mL_p^2}\right]^{2/3} S^{-2/3}\Lambda_n^{7/3}$	
Itoh (1993)	Inclusion of electron viscosity in Ohm's law.	$\chi_e = \frac{v_A}{Rq}\left(\frac{c}{\omega_{pe}}\right)^2 \left(\frac{\alpha^{3/2}}{h(s)}\right)$	$\alpha > m_i\beta_e/m_e$
Connor (1993)	As Itoh but transport due to stochastic magnetic field.	$\chi_e = \frac{v_{the}}{Rq}\left(\frac{c}{\omega_{pe}}\right)^2 \frac{\alpha^2}{s}$	$\alpha > m_i\beta_e/m_e$
Connor (1993)	Hot plasmas considered where electron Landau damping is important.	Turbulent convection $\chi_e = \frac{v_{the}}{Rq}\left(\frac{c}{\omega_{pe}}\right)^2 \alpha$ Stochastic magnetic field $\chi_e = \frac{v_{the}}{Rq}\left(\frac{c}{\omega_{pe}}\right)\rho_s \frac{\alpha^{3/2}}{s^{1/2}}$	$\alpha < m_i\beta_e/m_e$

Table A4. (Continued).

Author	Comments	D, χ_e	Conditions
Thayer (1987)	Resistivity-gradient driven turbulence with impurities. Suggested m value given in equation (5.3.21) or treat as a constant.	Particle diffusivity $D_n = \left(\frac{L_s J_z \eta}{L_\eta B}\right)^{4/3} (\chi_Z k_\parallel'^2)^{-1/3}(1+\eta_Z)^{4/3}$ Impurity diffusivity $D_Z = D_n$ Temperature diffusivity $D_T = (\chi_Z/\chi_T)^{1/3} D_n$	*Neglect of radiation effects:* $\Gamma_R \ll 1$ $\Gamma_Z \ll 1$ Threshold condition given by equation (5.3.1) but instability may be driven by nonlinear effects even when violated. $n_z Z \ll n_i$
Thayer (1990)	Radiative instability. Particle diffusivity. Suggested m value given in equation (5.3.21) or treat as a constant.	$D_n = \left(\frac{L_s J_z \eta}{L_\eta B}\right)^{4/3} (\chi_Z k_\parallel'^2)^{-1/3}$ $\times[\Gamma_T + (\chi_n/\chi_T)^{2/3}\Gamma_n]^2$	$\Gamma_n \gg 1$ or $\Gamma_T \gg 1$
Kwon(1989)	Resistivity-gradient mode using neoclassical MHD. Suggested m value given in equation (5.3.21) or treat as constant.	Temperature diffusivity: $D_t = \left[(C_n + C_t\eta_e)\left(\frac{cE_\parallel L_s}{BL_n}\right)\right]^{4/3} [\chi_\parallel k_\parallel'^2]^{-1/3}$ Density diffusivity: $D_n = \left[(C_n + C_t\eta_e)\left(\frac{cE_\parallel L_s}{BL_n}\right)\right]^{4/3} [\chi_{neo} k_\parallel'^2]^{-1/3}$	

References

Beklemishev A D and Horton W 1992 *Phys. Fluids* B **4** 200
Biglari H and Diamond P H 1991 *Phys. Fluids* B **3** 1797
Biglari H, Diamond P H and Rosenbluth M N 1989 *Phys. Fluids* B **1** 109
Biglari H, Diamond P H and Terry P W 1988 *Phys. Fluids* **31** 2644; *Phys. Rev. Lett.* **60** 200
Bishop C M and Connor J W 1990 *Plasma Phys. Control. Fusion* **32** 203
Callen J D and Shaing K C 1985 *Phys. Fluids* **28** 1845
Carreras B A and Diamond P H 1983b *Phys. Rev. Lett.* **50** 503
——1989 *Phys. Fluids* B **1** 1011
Carrera R, Hazeltine R D and Kotschenreuther M 1986 *Phys. Fluids* **29** 899
Carreras B A, Garcia L and Diamond P H 1987 *Phys. Fluids* **30** 1388
Carreras B A *et al* 1992 *Phys. Fluids* B **4** 3115
Chen L and Cheng C Z 1980 *Phys. Fluids* **23** 2242
Cohen B I *et al* 1976 *Nucl. Fusion* **16** 971
Connor J W 1986a *Nucl. Fusion* **26** 193
——1986b *Nucl. Fusion* **26** 517
——1988 *Plasma Phys. Control. Fusion* **30** 619
——1993 *Plasma Phys. Control. Fusion* **35** 757
Connor J W and Chen L 1985 *Phys. Fluids* **28** 2207
Connor J W and Taylor J B 1984 *Phys. Fluids* **27** 2676
——1985 *Proc. 10th Int. Conf. Plasma Physics and Controlled Thermonuclear Research (London 1984)* Vol II (IAEA Vienna) p 13
Connor J W, Taylor J B and Wilson H R 1993 *Phys. Rev. Lett.* **70** 1803
Connor J W *et al* 1993 *Plasma Phys. Control. Fusion* **35** 319
Diamond P H and Biglari H 1990 *Phys. Rev. Lett.* **65** 2865
Diamond P H and Rosenbluth M N 1981 *Phys. Fluids* **24** 1641
Diamond P H *et al* 1983 *Proc. 9th Int. Conf. Plasma Physics and Controlled Thermonuclear Research (Baltimore 1982)* vol I (IAEA Vienna) p259
——1991 *Proc. 13th Int. Conf. Plasma Physics and Controlled Thermonuclear Research (Washington 1990)* vol II (IAEA Vienna) p 9
Dominguez R R 1990 *Phys. Fluids* B **2** 2151
Dominguez R R and Waltz R E 1987 *Nucl. Fusion* **27** 65
——1988 *Phys. Fluids* **31** 3147
——1989 *Nucl. Fusion* **29** 885
Dubois M A and Mohamed-Benkadda M S 1991 *CEA/Euratom report* (Cadarache) *EUR–CEA–FC 1434*
Gang F Y, Diamond P H and Rosenbluth M N 1991 *Phys. Fluids* B **3** 68
Garbet X, Mourgues F and Samain A 1990a *Plasma Phys. Control. Fusion* **32** 131
——1990b *Plasma Phys. Control. Fusion* **32** 917
Garbet X *et al* 1992 *Phys. Fluids* B **4** 136
Garcia L *et al* 1985 *Phys. Fluids* **28** 2147
Guo S C and Romanelli F 1993 *Phys. Fluids* B **5** 520
Guo S C *et al* 1989 *Plasma Phys. Control. Fusion* **31** 423
Guzdar P N *et al* 1983 *Phys. Fluids* **26** 673
——1986 *Phys. Rev. Lett.* **57** 2818
Hagan W K and Frieman E A 1986 *Phys. Fluids* **29** 3635
Hahm T S and Diamond P H 1987 *Phys. Fluids* **30** 133
Hahm T S and Tang W M 1989 *Phys. Fluids* B **1** 1185
——1991 *Phys. Fluids* B **3** 989
Hahm T S *et al* 1987 *Phys. Fluids* **30** 1452
Hamaguchi S and Horton W 1990 *Phys. Fluids* B **2** 1833
Hassam A B 1980 *Phys. Fluids* **23** 38
Hassam A B and Drake J F 1983 *Phys. Fluids* **26** 133
Hassam A B *et al* 1990 *Phys. Fluids* B **2** 1822
Hirshman S P and Molvig K 1979 *Phys. Rev. Lett.* **42** 648
Hong B G, Choi D and Horton W 1986 *Phys. Fluids* **29** 1872
Hong B G and Horton W 1990 *Phys. Fluids* B **2** 978
Horton W 1976 *Phys. Fluids* **19** 711

——1985 *Plasma Phys. Control. Fusion* **27** 937
——1990 *Phys. Rep.* **192** 1
Horton W, Choi D and Tang W M 1981 *Phys. Fluids* **24** 1077
Horton W, Choi D, Yushmanov P N and Parail V V 1987 *Plasma Phys. Control. Fusion* **29** 901
Horton W, Hong B G and Tang W M 1988 *Phys. Fluids* **31** 2971
Horton W and Varma R K 1972 *Phys. Fluids* **15** 620
Hua D D, Xu X Q and Fowler T K 1992 *Phys. Fluids* B **4** 3216
Hugon M and Rebut P-H 1991 *Proc. 13th Int. Conf. Plasma Physics and Controlled Nuclear Fusion Research (Washington 1990)* vol II (IAEA Vienna) p 45
Itoh K *et al* 1993 *Plasma Phys. Control. Fusion* **35** 543
Kadomtsev B B 1991 *Nucl. Fusion* **31** 1301
——1992 *Plasma Phys. Control. Fusion* **34** 1931
Kadomtsev B B and Pogutse O P 1970 *Rev. Plasma Phys.* vol 5 ed M A Leontovich (New York: Consultants Bureau) p 249
——1971 *Nucl. Fusion* **11** 67
——1985 *Proc. 10th Int. Conf. on Plasma Physics and Controlled Nuclear Fusion Research (London 1984)* vol II (IAEA Vienna) p 69
Kaw P K 1982 *Phys. Lett.* **90A** 290
Kesner J 1989 *Nucl. Fusion* **29** 1397
Kim D E *et al* 1990 *Phys. Fluids* B **2** 547
Kim Y B *et al* 1991 *Phys. Fluids* B **3** 384
Kotschenreuther M 1988 *University of Texas report* IFSR 319
——1991 *Proc. 13th Int. Conf. Plasma Physics and Controlled Thermonuclear Research (Washington 1990)* vol II (IAEA Vienna) p 361
Kotschenreuther M, Hazeltine R D and Morrison P J 1985 *Phys. Fluids* **28** 294
Krommes J A and Kim C-B, 1988 *Phys. Fluids* **31** 869
Kuvshinov B N *et al* 1989 *Sov. J. Plasma Phys.* **15** 671
Kwon O J, Diamond P H and Biglari H 1990 *Phys. Fluids* B **2** 291
Kwon O J, Diamond P H and Hahm T S 1989 *Phys. Fluids* B **1** 2172
Lallia P P, Rebut P-H and Watkins M L, 1988 *JET Report* JET-P(88)05
Leboeuf J N *et al* 1991 *Phys. Fluids* B **3** 2291
Lee G S and Diamond P H 1986 *Phys. Fluids* **29** 3291
Lee Y C *et al* 1987 *Phys. Fluids* **30** 1331
Liewer P 1985 *Nucl. Fusion* **25** 543
Manheimer W M and Antonsen T M 1979 *Phys. Fluids* **22** 957
Manheimer W M *et al* 1976 *Phys. Rev. Lett.* **37** 287
Mattor N 1989 *Culham Fusion Theory Preprint* PPN 89/20
——1991 *Phys. Fluids* B **3** 2153
Mattor N and Diamond P H 1989 *Phys. Fluids* B **1** 1980
Mikhailovskii A B 1976 *JETP Lett.* **23** 394
Molvig K, Hirshman S P and Whitson J C 1979 *Phys. Rev. Lett.* **43** 582
Nordman H, Weiland J and Jarmén A 1990 *Nucl. Fusion* **30** 983
Ohkawa T 1978 *Phys. Lett.* **67A** 35
Parail V V and Pogutse O P 1981 *JETP Lett.* **32** 384; *Proc. 8th Int. Conf. Plasma Physics and Controlled Nuclear Fusion Research (Brussels 1980)* vol I (IAEA Vienna) p 67
Parail V V and Yushmanov P N 1985 *JETP Lett.* **42** 343
Pearlstein L D and Berk H L 1969 *Phys. Rev. Lett.* **23** 220
Perkins F W 1984 *Proc. of the 4th Int. Symp. Heating in Toroidal Plasmas (ENEA Rome 1984)* vol 2 p 977
Rebut P-H and Hugon M 1985 *Proc. 10th Int. Conf. Plasma Physics and Controlled Nuclear Fusion Research (London 1984)* vol II (IAEA Vienna) p 197
——1991 *Plasma Phys. Control. Fusion* **33** 1085
Rebut P H, Lallia P P and Watkins M L 1989 *Proc. 12th Int. Conf. Plasma Physics and Controlled Thermonuclear Research (Nice 1988)* vol II (IAEA Vienna) p191
Rechester A B and Rosenbluth M N 1978 *Phys. Rev. Lett.* **40** 88
Rogister A 1989 *Proc. Cadarache Workshop on Electrostatic Turbulence 1989* EUR-CEA-FC-1381
Rogister A, Hasselberg G, Waelbroek F G and Weiland J 1988 *Nucl. Fusion* **28** 1053
Romanelli F 1989 *Phys. Fluids* B **1** 1018
Romanelli F and Briguglio S 1990 *Phys. Fluids* B **2** 754

Romanelli F, Chen L and Briguglio S 1991 *Phys. Fluids* B **3** 2496
Romanelli F, Tang W M and White R B 1986 *Nucl. Fusion* **26** 1515
Romanelli F and Zonca F 1993 *20th EPS Conf. Controlled Fusion and Plasma Physics (Lisboa 1993)* Part IV p 1387
Ross D W and Mahajan S M 1978 *Phys. Rev. Lett.* **40** 324
Ross D W *et al* 1987 *University of Texas Report* FRCR-295
Rozhanskii V A 1981 *JETP Lett.* 34 56
Rutherford P H 1973 *Phys. Fluids* **16** 1903
Samain A 1984 *Plasma Phys. Control. Fusion* **26** 731
Similon P L and Diamond P H 1984 *Phys. Fluids* **27** 916
Smolyakov A I 1989 *Sov. J. Plasma Phys.* **15** 667
——1993 *Plasma Phys. Control. Fusion* **35** 657
Smolyakov A I and Hirose A 1993 *Phys. Fluids* B **5** 663
Strauss H R 1981, *Phys. Fluids* **24** 2004
Sydora R D *et al* 1990 *Phys. Rev. Lett.* **64** 2015
Tang W M 1986 *Nucl. Fusion* **26** 1605
——1990 *Theory of Fusion Plasmas* E Sindoni, F Troyon and J Vaclavik (Varenna 1990) (Bologna: Editrice Compositori) 31
Tang W M, Romanelli F and Briguglio S 1988 *Phys. Fluids* **31** 2951
Tang W M *et al* 1975 *Phys. Rev. Lett.* **35** 660
Taroni A *et al* 1991 *Proc. 13th Int. Conf. Plasma Physics and Controlled Nuclear Fusion Research (Washington 1990)* vol I (IAEA Vienna) p 93
Taylor J B 1977 *Proc. 6th Int. Conf. Plasma Physics and Controlled Thermonuclear Research (Berchtesgaden 1976)* vol II (IAEA Vienna) p 323
Terry P W and Diamond P H 1983 *University of Texas Report* IFSR 91
Terry P W, Diamond P H and Hahm T S 1986 *Phys. Rev. Lett.* **57** 1899
Terry P W *et al* 1988 *Phys. Fluids* **31** 2920
Thayer D R and Diamond P H 1987 *Phys. Fluids* **30** 3724
——1990 *Phys. Rev. Lett.* **65** 2784
Tibone F, Connor J W, Stringer T E and Wilson H R 1994 *Plasma Phys. Control. Fusion* **36** 473
Tsang K T, Catto P J, Whitson J C and Smith J 1978 *Phys. Rev. Lett.* **40** 327
Waltz R E 1982 *Phys. Fluids* **25** 1269
Waltz R E *et al* 1987 *Proc. 11th Int. Conf. Plasma Physics and Controlled Thermonuclear Research (Kyoto 1986)* vol I (IAEA Vienna) p 345
——1986 *Nucl. Fusion* **26** 1729
Weiland J and Nordman H 1991 *Nucl. Fusion* **31** 390
White R B and Romanelli F 1989 *Phys. Fluids* B **1** 977
Xu X Q and Rosenbluth M N 1990 *University of Texes Report* IFSR #443
——1991 *Phys. Fluids* B **3** 627
Zhang Y Z and Mahajan S M 1988 *Comments Plasma Phys. Control. Fusion* **11** 243